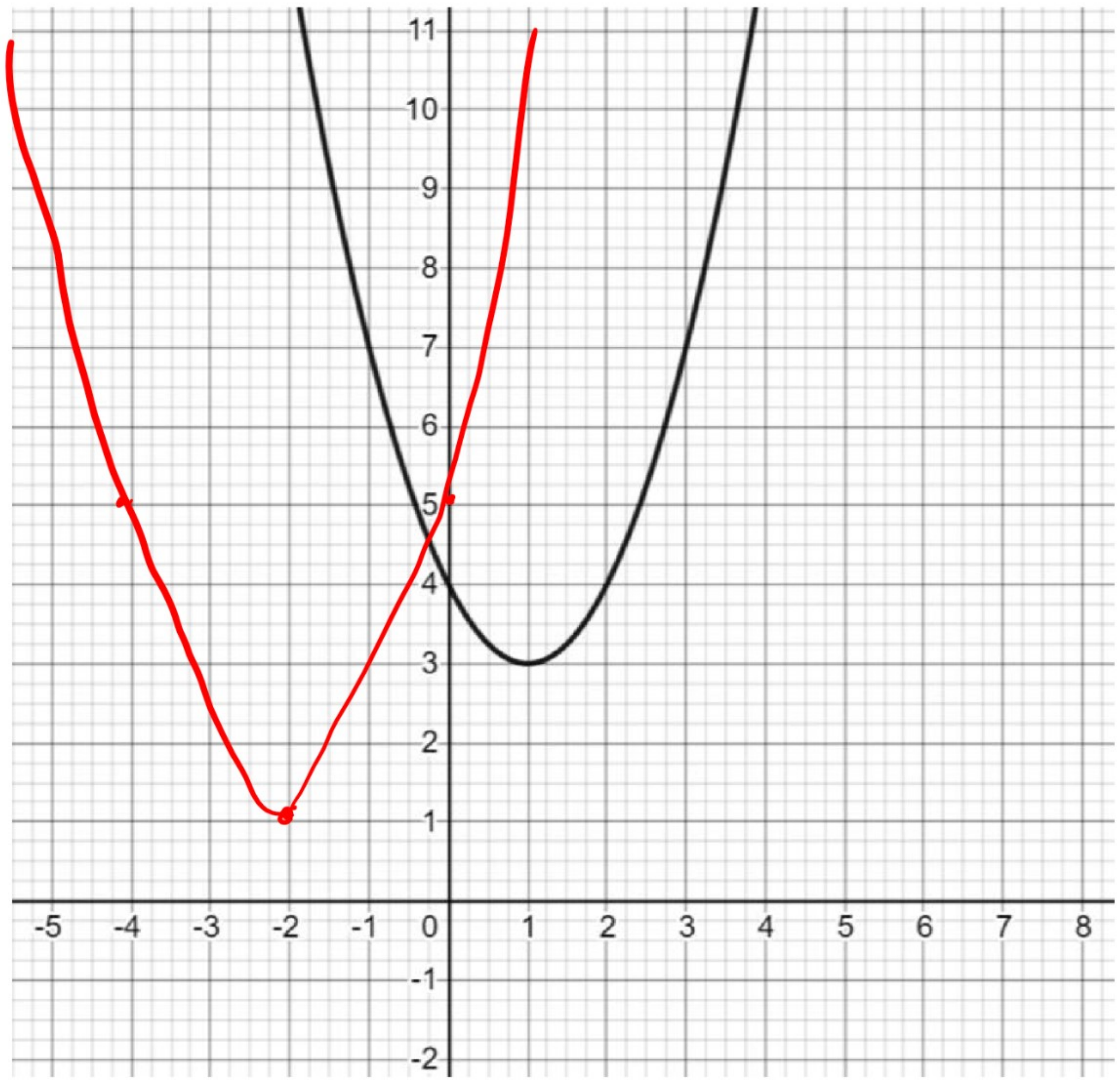
Transforming Graphs of Functions Exam Practice

Q1. The graph below shows a sketch of $y = f(x)$. On the grid, draw the graph $y = f(x+3) - 2$.

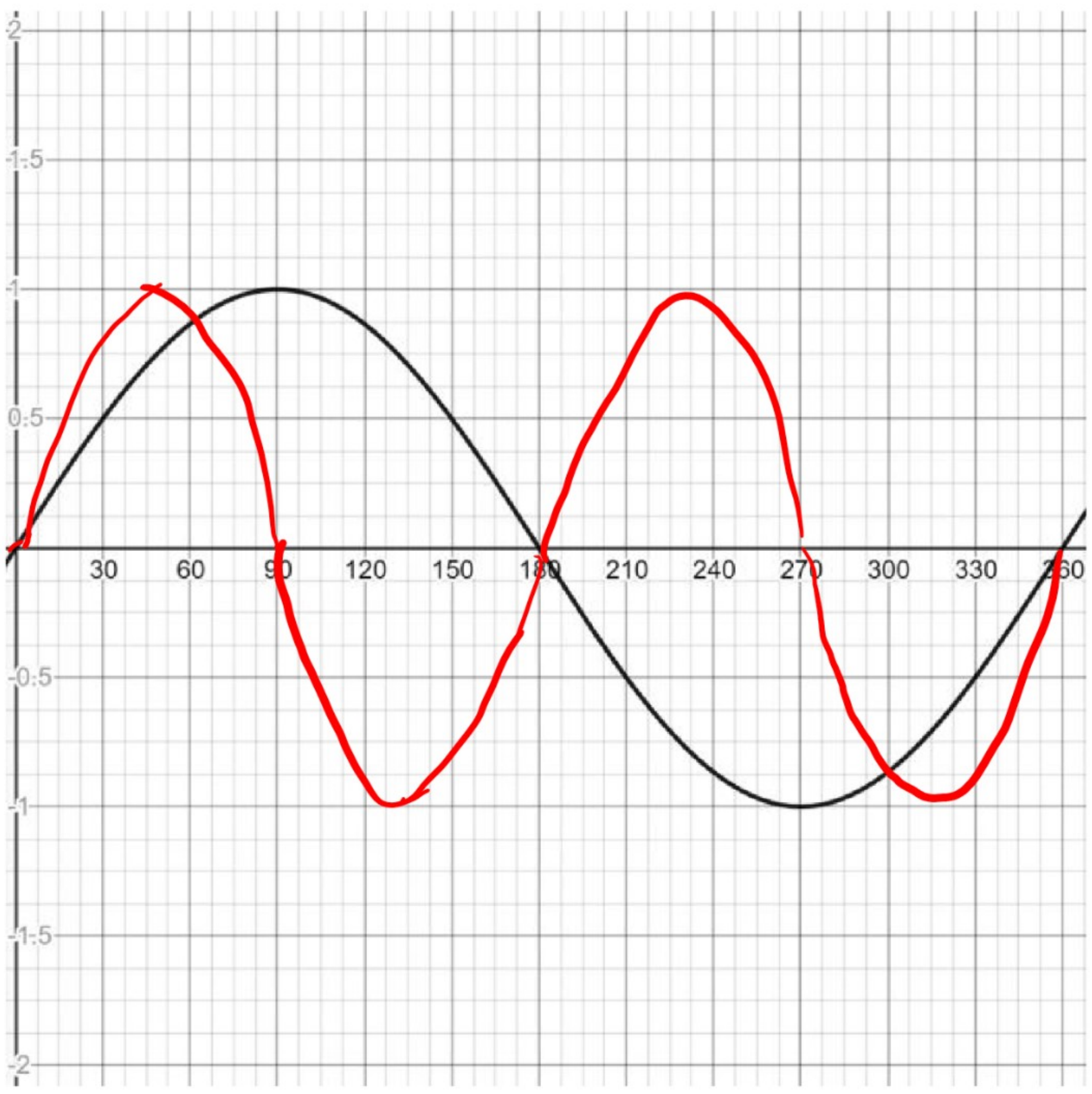


This is a translation by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Answer:____________________

(2 marks)

Q2. The graph below shows a sketch of $y = f(x)$. On the grid, draw the graph $y = f(2x)$.

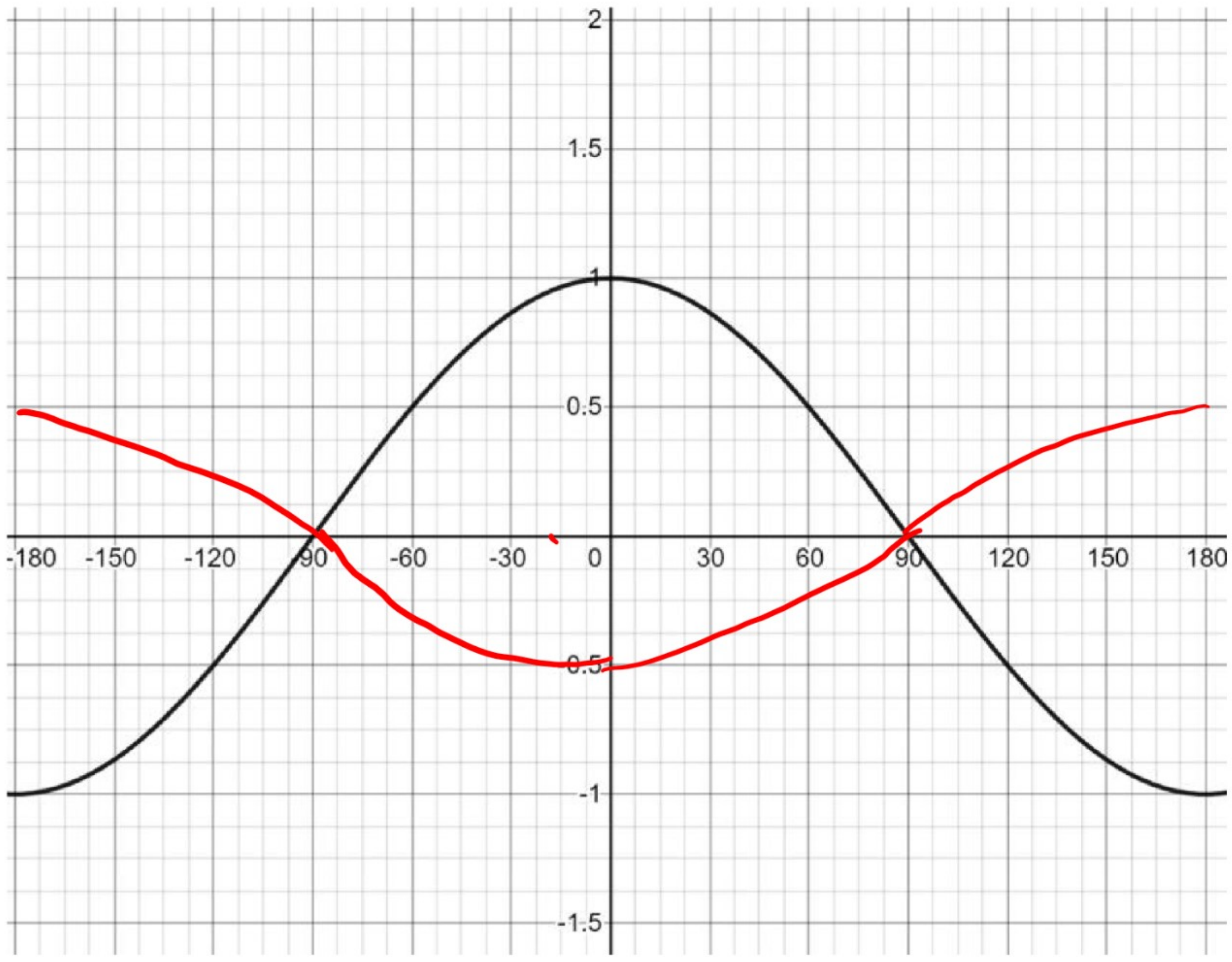


This is a stretch, scale factor $\frac{1}{2}$, in the x-direction.

Answer:____________________

(2 marks)

Q3. The graph below shows a sketch of y = f(x). On the grid, draw the graph $y = -\frac{1}{2}f(x)$

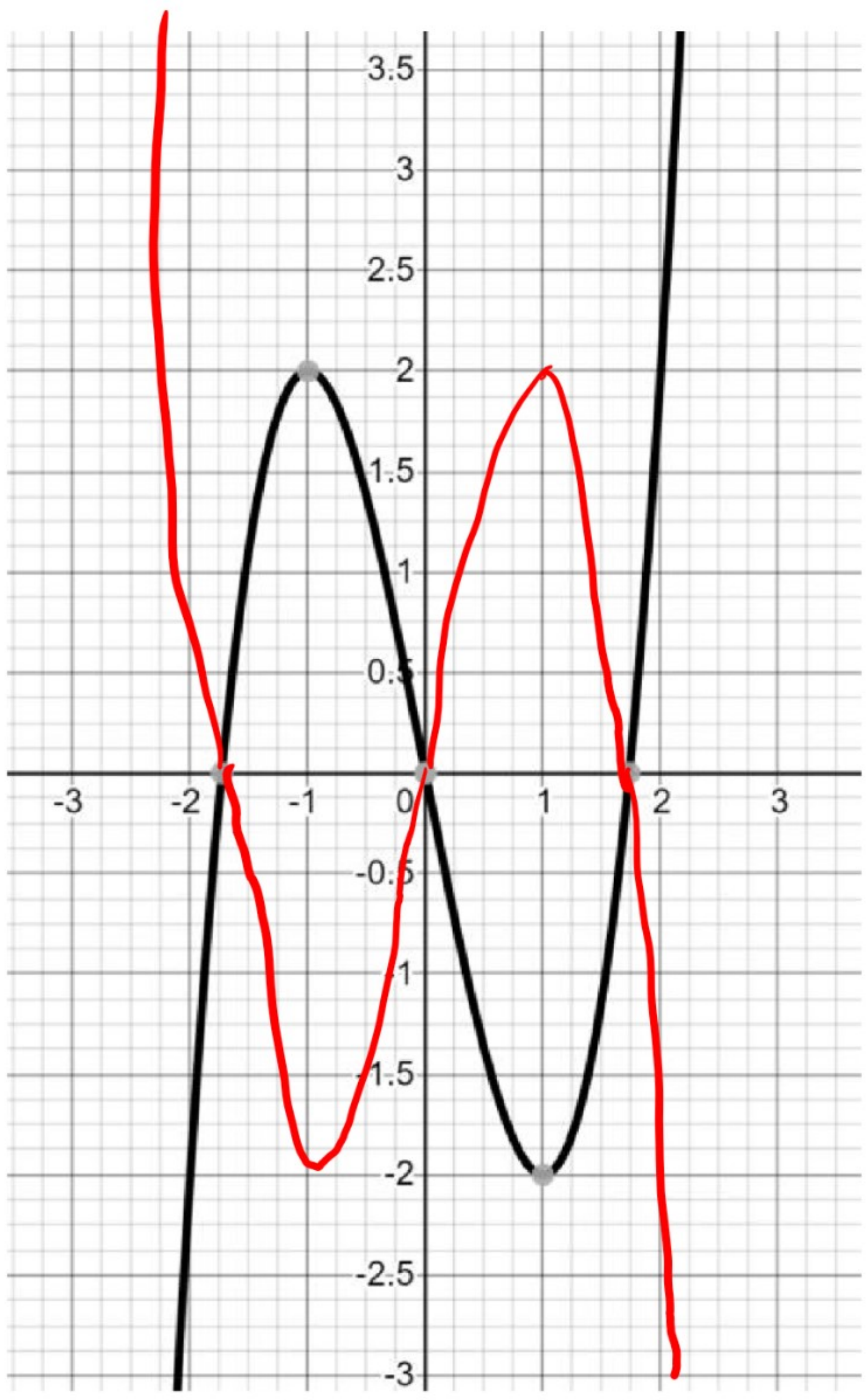


This is a stretch scale factor $-\frac{1}{2}$ in the y-direction

Answer:____________________

(2 marks)

Q4. The graph below shows a sketch of y = f(x). On the grid, draw the graph y = f(-x).

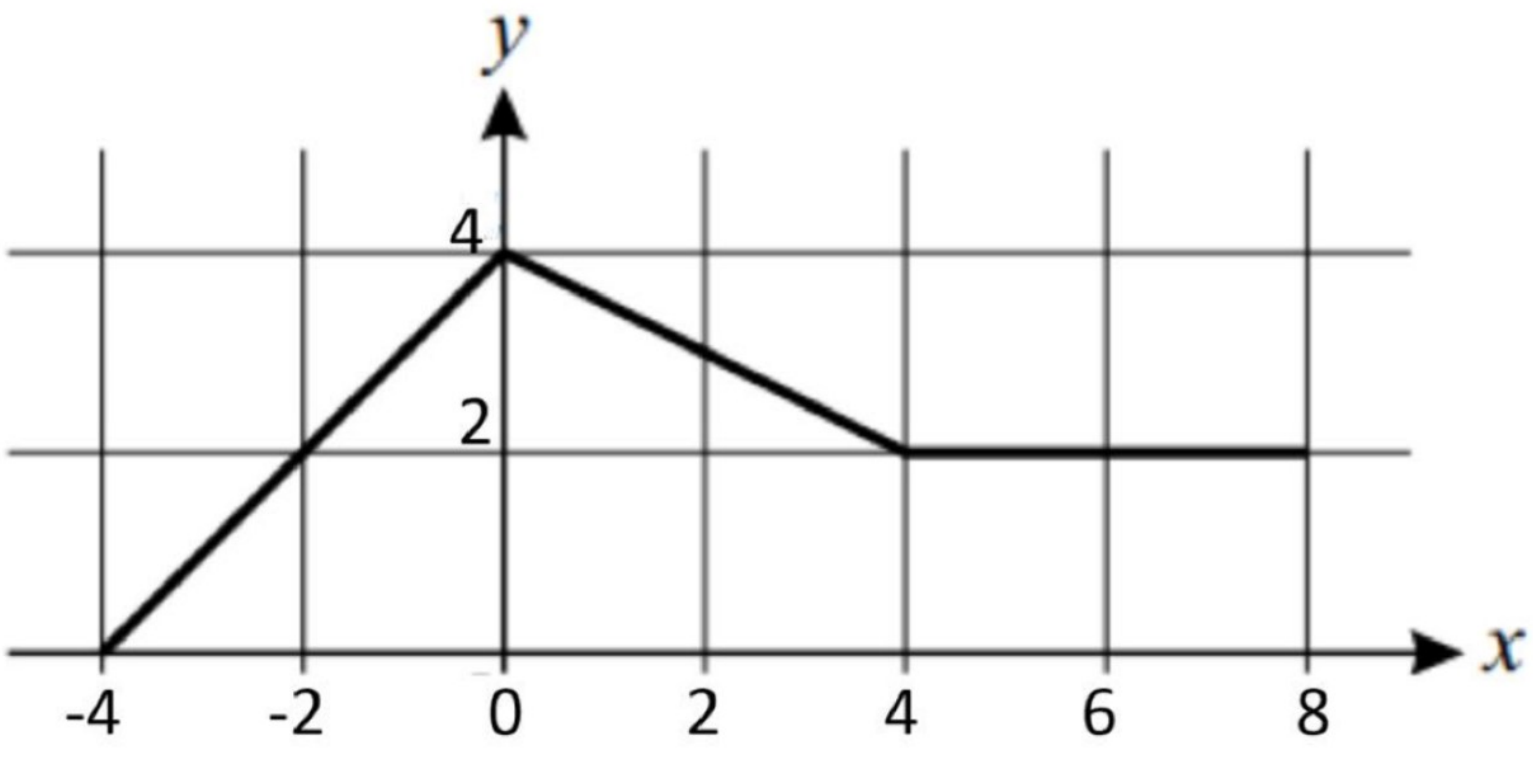


This is a reflection in the y-axis

Answer:____________________

(2 marks)

Q5. The graph below is a sketch of y = f(x) which is defined for $-4 \leq x \leq 8$.



a) Write down the value of f(5.5)

Answer: 2

(1 mark)

b) Let g(x) = f(-x). Find the value of g(-2).

$g(-2) = f(2)$
$= 3.5$

Answer: 3.5

(1 mark)

c) Let h(x) be such that h(-4) = 0 & h(4) = 6.
Describe fully a possible transformation which takes f(x) to h(x).

Stretch, scale factor 3, in the y-direction

Answer:

(3 marks)

Q6. Let $f(x) = 2x^2 + 4x - 5$. Describe fully the single transformation which takes $f(x)$ to each of the following graphs.

(i) $g(x) = 2x^2 + 4x + 7$ Translation by $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$

(ii) $h(x) = 2x^2 - 4x + 7$ Reflection in y-axis $(h(x) = f(-x))$

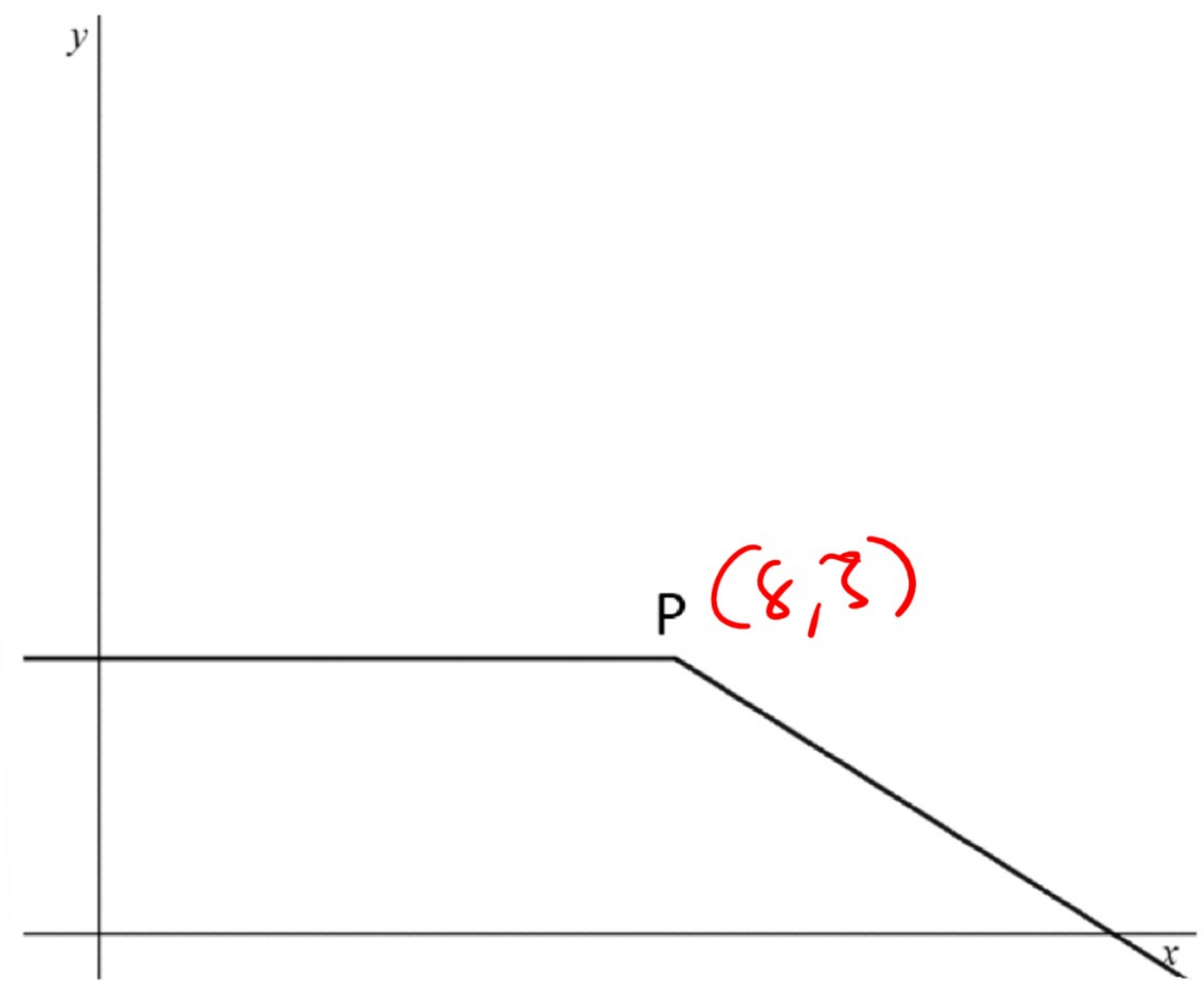
(iii) $k(x) = 8x^2 + 8x - 5$ $k(x) = f(2x)$, so stretch scale factor $\frac{1}{2}$ in the x-direction

Answer:____________________

(6 marks)

Q7. Let f(x) be the graph below. The vertex P has coordinates (8, 3).

a) Work out the coordinates of the vertex in each of the following cases:



(i) $g(x) = f(x - 4)$ (ii) $g(x) = f(\frac{1}{3}x)$ (iii) $g(x) = -2f(x)$

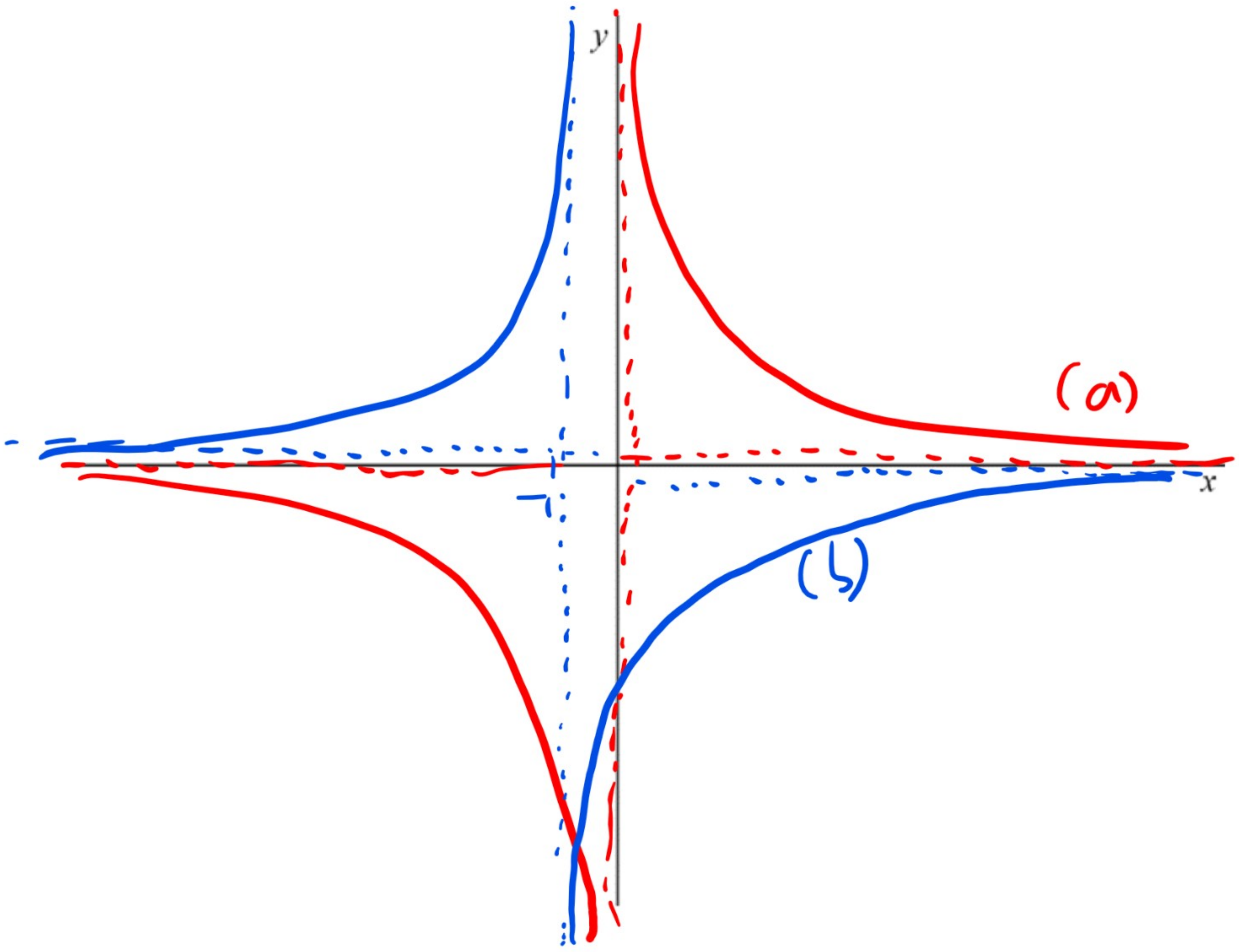
(12,3) (24,3) (8,−6)

Answer:____________________

(6 marks)

Q8. On the grid below sketch the following graphs, clearly indicating any asymptotes:

a) $y = \frac{1}{x}$ b) $y = -\frac{1}{x+1}$



Answer:______________________

(6 marks)

Q9. The graph of $y = 5^x$ can be transformed into the graph of $y = 5^{x-2}$ by two different transformations.
Describe each of these transformations fully.

(i) $y = 5^{-2}5^{x}$, $\Rightarrow$ stretch scale factor $\frac{1}{25}$ in the y-direction

or

(ii) if $f(x) = 5^x$, $f(x-2) = 5^{x-2}$ $\Rightarrow$ translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Answer:______________________

(4 marks)

Q10. Let $f_0(x) = \sin(x)$,

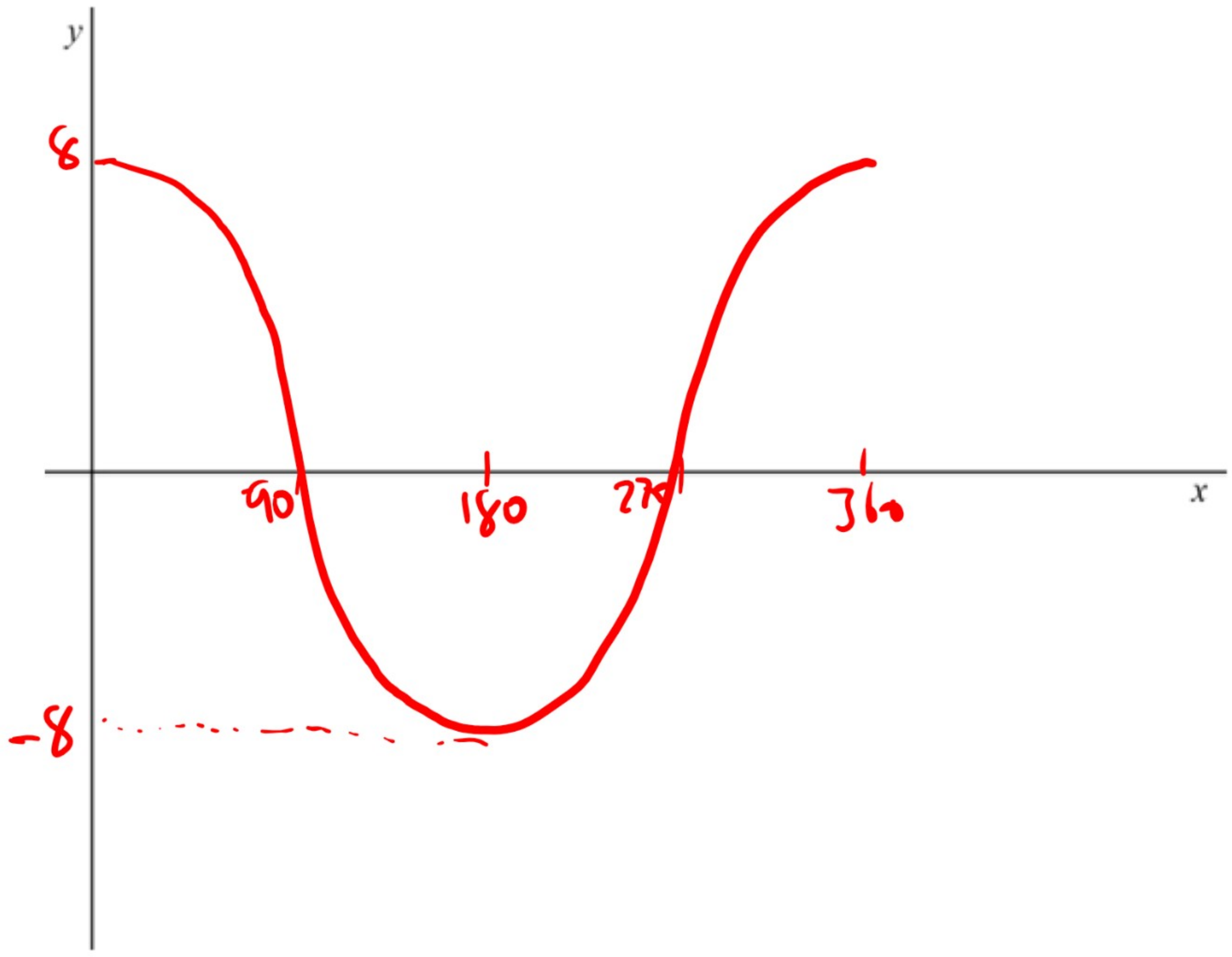
$$f_{n+1}(x) = 2f_n(x + 30°)$$

be an iteration formula for a sequence of functions.

a) Sketch $f_3(x)$ on the axes for $0 \leq x \leq 360°$

$f_1 = 2\sin(x+30)$, $f_2 = 4\sin(x+60)$, $f_3 = 8\sin(x+90)$

$(= 8\cos(x))$



Answer:____________________

(4 marks)

b) Solve the equation $f_{12}(x) = 1000$ for $0 \leq x \leq 90°$ to 1 d.p.

$f_{12}(x) = 2^{12}\sin(x+360)$

$= 2^{12}\sin(x)$

$\Rightarrow 2^{12}\sin(x) = 1000$

$\sin(x) = \frac{1000}{4096}$

$x = 14.1°$

Answer: 14.1°

(3 marks)

Q11. Let $f(x) = 4x^3 + 10$. Determine the function g(x) which $f(x)$ is mapped onto in each of the following cases:

(i) translation by the vector $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

$g(x) = f(x+3) - 4$

$= 4(x+3)^3 + 10 - 4$

$\Rightarrow g(x) = 4(x+3)^3 + 6$

Answer: $g(x) = 4(x+3)^3 + 6$

(2 marks)

(ii) reflection in the $y - axis$

$g(x) = f(-x)$

$g(x) = 4(-x)^3 + 10$

$g(x) = -4x^3 + 10$

Answer: $g(x) = -4x^3 + 10$

(2 marks)

(iii) stretch in the x −direction scale factor $\frac{1}{3}$

$g(x) = f(3x)$

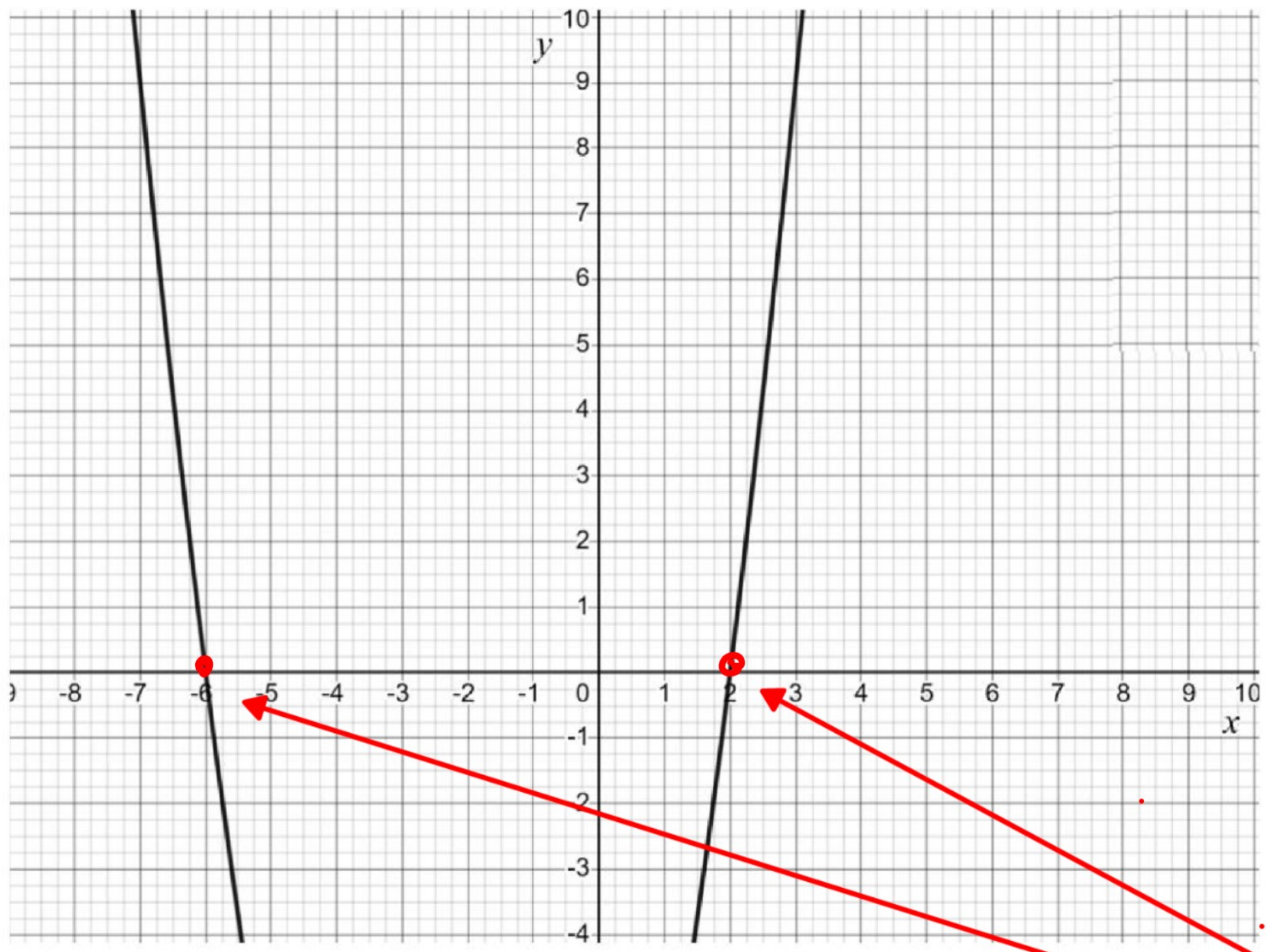
$g(x) = 4(3x)^3 + 10$

$g(x) = 108x^3 + 10$

Answer: $g(x) = 108x^3 + 10$

(2 marks)

Q12. Below is part of a quadratic graph $y = f(x)$, which has turning point P.
The transformed graph $g(x) = f(2x) + 25$ has turning point Q.
The y coordinate of Q is 9. Find the full co-ordinates of P and Q.



- x coordinate of P is $\frac{1}{2}(-6+2)$ is -2 using symmetry of a quadratic graph.
- x coordinate of Q is -1 applying stretch, scale factor 2 in x-direction as $f(2x)$
- y coordinate of Q is 9 $\Rightarrow$ y-coordinate of P is $9 - 25 = -16$ reversing the translation $\begin{pmatrix} 0 \\ 25 \end{pmatrix}$

$\Rightarrow$ P$(-2,-16)$, Q$(-1,7)$

Answer: P$(-2,-16)$, Q$(-1,7)$

(4 marks)