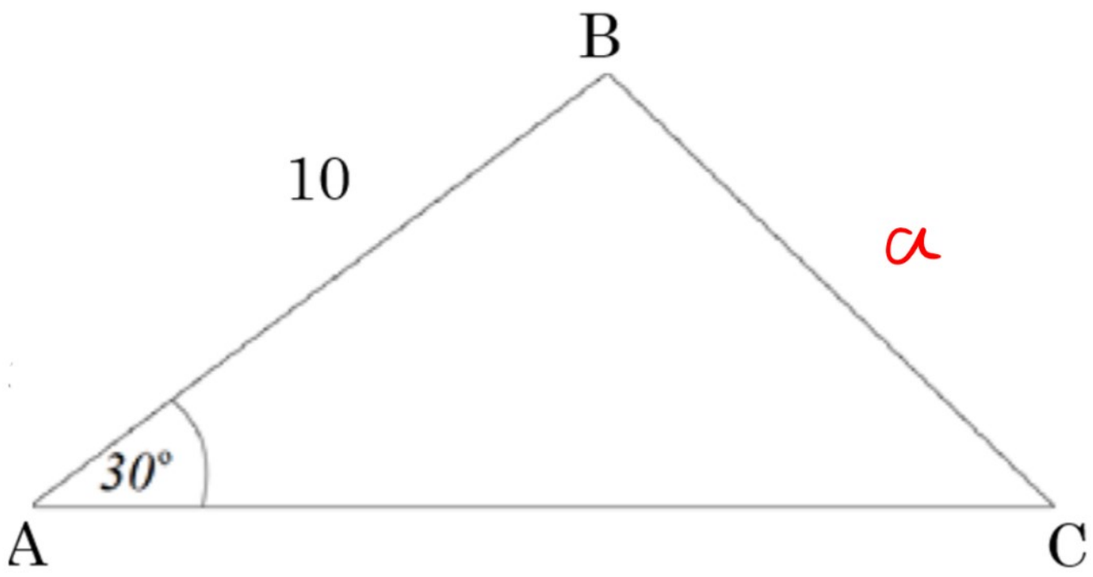
The Cosine Rule Exam Practice

Q1. Find the length of side BC in the triangle below to 1 decimal place.



$a^2 = 5^2 + c^2 - 2bc\cos A$

$a^2 = 12^2 + 10^2 - 2(12)(10)\cos(30)$

$a^2 = 36.15$

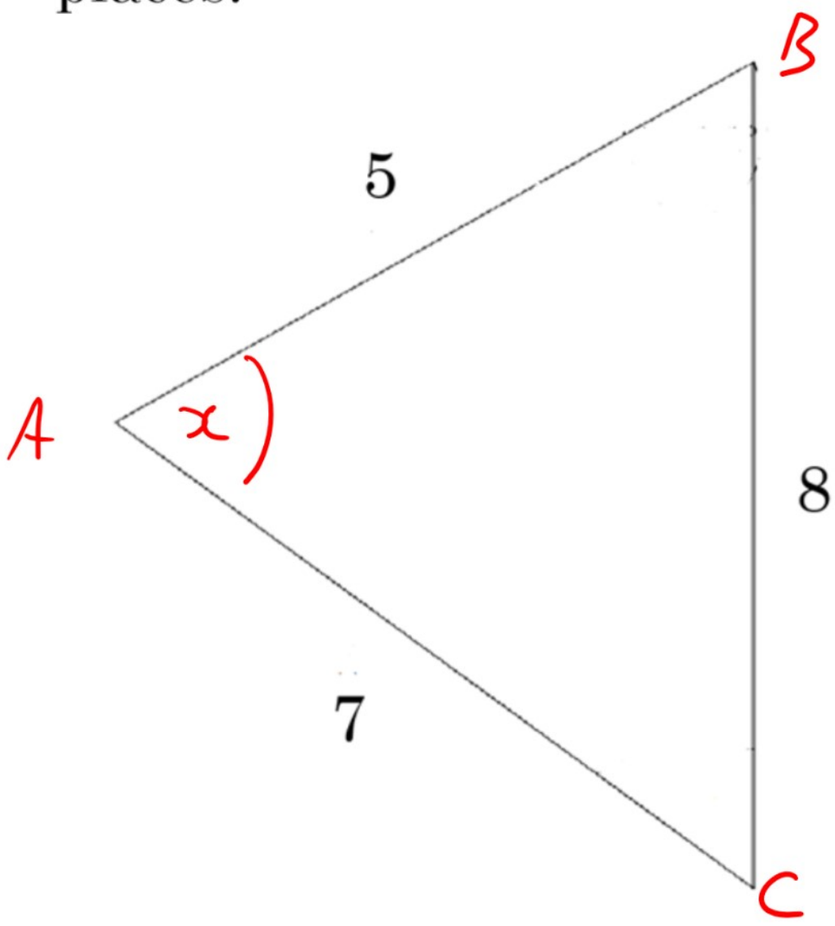
$a = \sqrt{36.15}$

$a = 6.0$

Answer: 6.0

(3 marks)

Q2. Find the size of the largest angle in the triangle below to 2 decimal places.



$$a^2 = b^2 + c^2 - 2(b)(c)\cos A$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{7^2 + 5^2 - 8^2}{2(7)(5)}$$

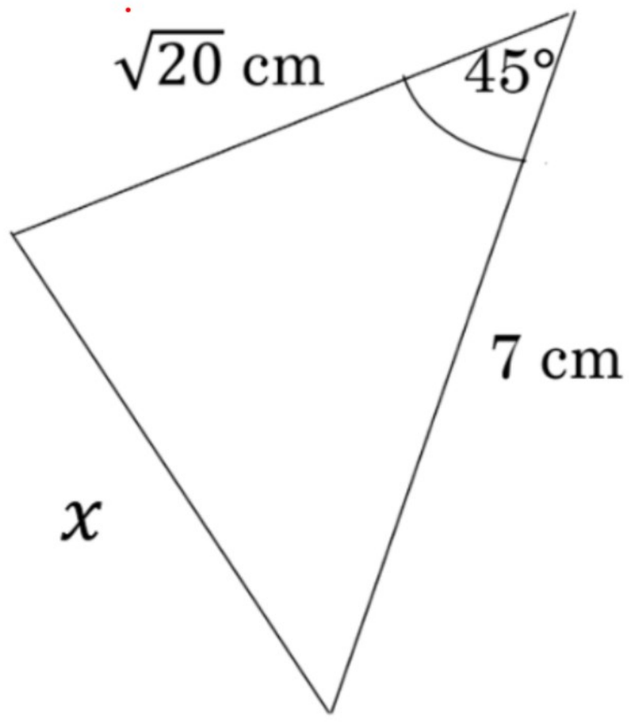
$$A = \cos^{-1}\left(\frac{1}{7}\right)$$

$$A = 81.8^\circ$$

Answer: 81.8°

(4 marks)

Q3. Find an expression for x^2 where x is the length shown in the triangle below, giving your answer in the form $a + b\sqrt{c}$, where a, b and c are integers.



$x^2 = 7^2 + (\sqrt{20})^2 - 2(7)\sqrt{20}\cos(45°)$

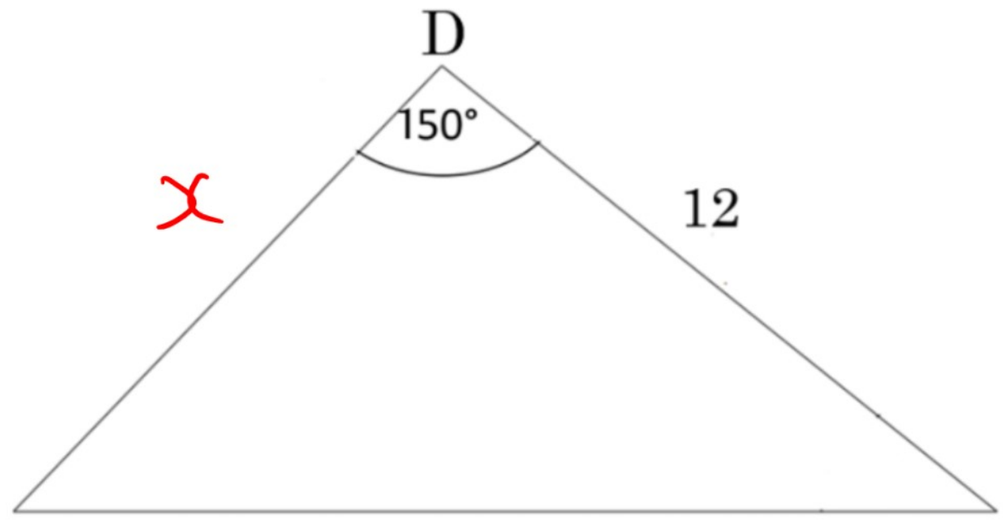
$x^2 = 69 - \frac{14\sqrt{20}}{\sqrt{2}}$

$x^2 = 69 - 14\sqrt{10}$

Answer: $69 - 14\sqrt{10}$

(4 marks)

Q4. The area of the triangle below is 30 squared units.



(i) Find the length of side DE to 1 d.p.

$\text{Area} = \frac{1}{2}ab\sin C$

$30 = \frac{1}{2}x(12)\sin(150)$

$x = 10$

Answer: 10

(3 marks)

(ii) Find the perimeter of the triangle to 3 s.f.

use $a^2 = b^2 + c^2 - 2bc\cos A$ to find EF

$(EF)^2 = 10^2 + 12^2 - 2(10)(12)\cos(150)$

$EF = 21.256\ldots$

Perimeter $= 10 + 12 + 21.256\ldots$

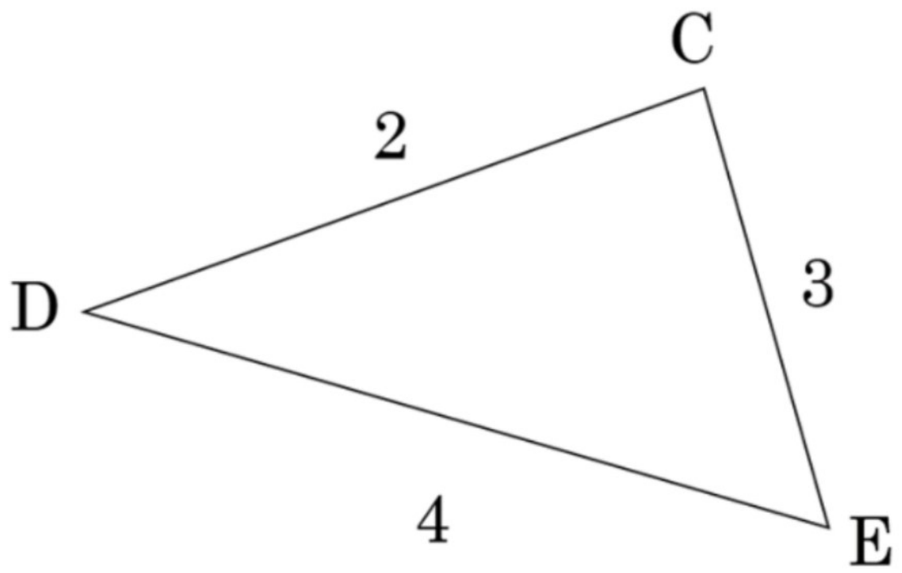
$= 43.256\ldots$

$= 43.3$ (3 s.f.)

Answer: 43.3

(3 marks)

Q5.



(i) Show clearly that $\cos(E) = \frac{7}{8}$

$2^2 = 3^2 + 4^2 - 2(3)(4)\cos(E)$

$\cos(E) = \frac{3^2+4^2-2^2}{2(3)(4)}$

$\cos(E) = \frac{7}{8}$

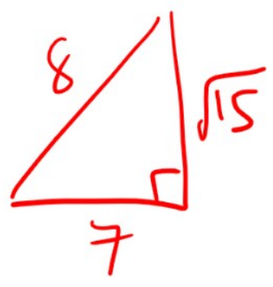
Answer: _____

(4 marks)

(ii) Hence show that the exact area of the triangle is $\frac{3}{4}\sqrt{15}$.

using $\frac{1}{2}bc\sin A$,

$\text{area} = \frac{1}{2}(4)(3)\sin(E)$



$\Rightarrow \sin(E) = \frac{\sqrt{15}}{8}$

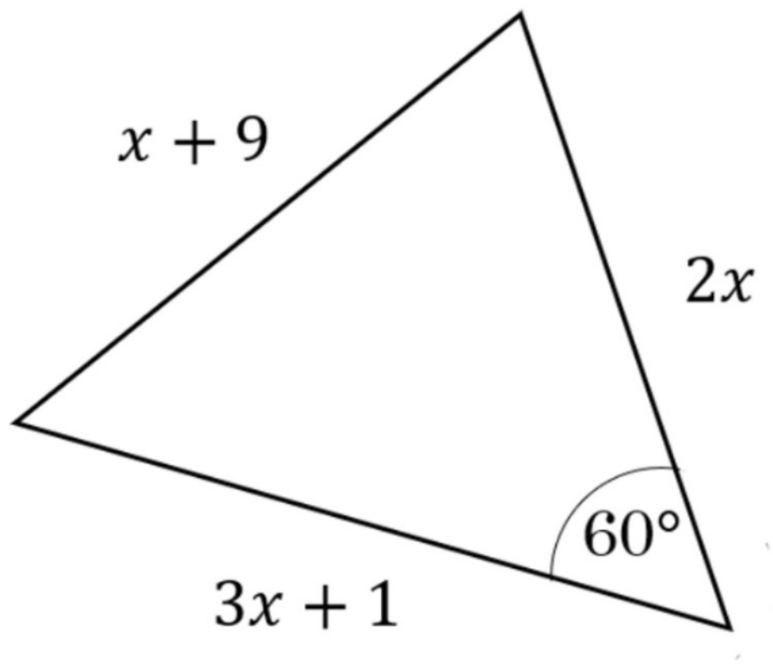
$\text{area} = \frac{1}{2}(4)(3)\frac{\sqrt{15}}{8}$

$= \frac{3\sqrt{15}}{4}$

Answer: _____

(3 marks)

Q6. Find the area of the triangle below:



- First find the value of x.

$$(x+9)^2 = (2x)^2 + (3x+1)^2 - 2(2x)(3x+1)\cos 60$$

$$x^2 + 18x + 81 = 4x^2 + 9x^2 + 6x + 1 - 6x^2 - 2x$$

$$0 = 6x^2 - 14x - 80$$

$$0 = 3x^2 - 7x - 40$$

$$0 = (3x + 8)(x - 5)$$

$$\Rightarrow x = -\frac{8}{3}, 5$$

$$(x) \quad (\checkmark)$$

- $\text{Area} = \frac{1}{2}(3(5)+1)(2(5))\sin 60$

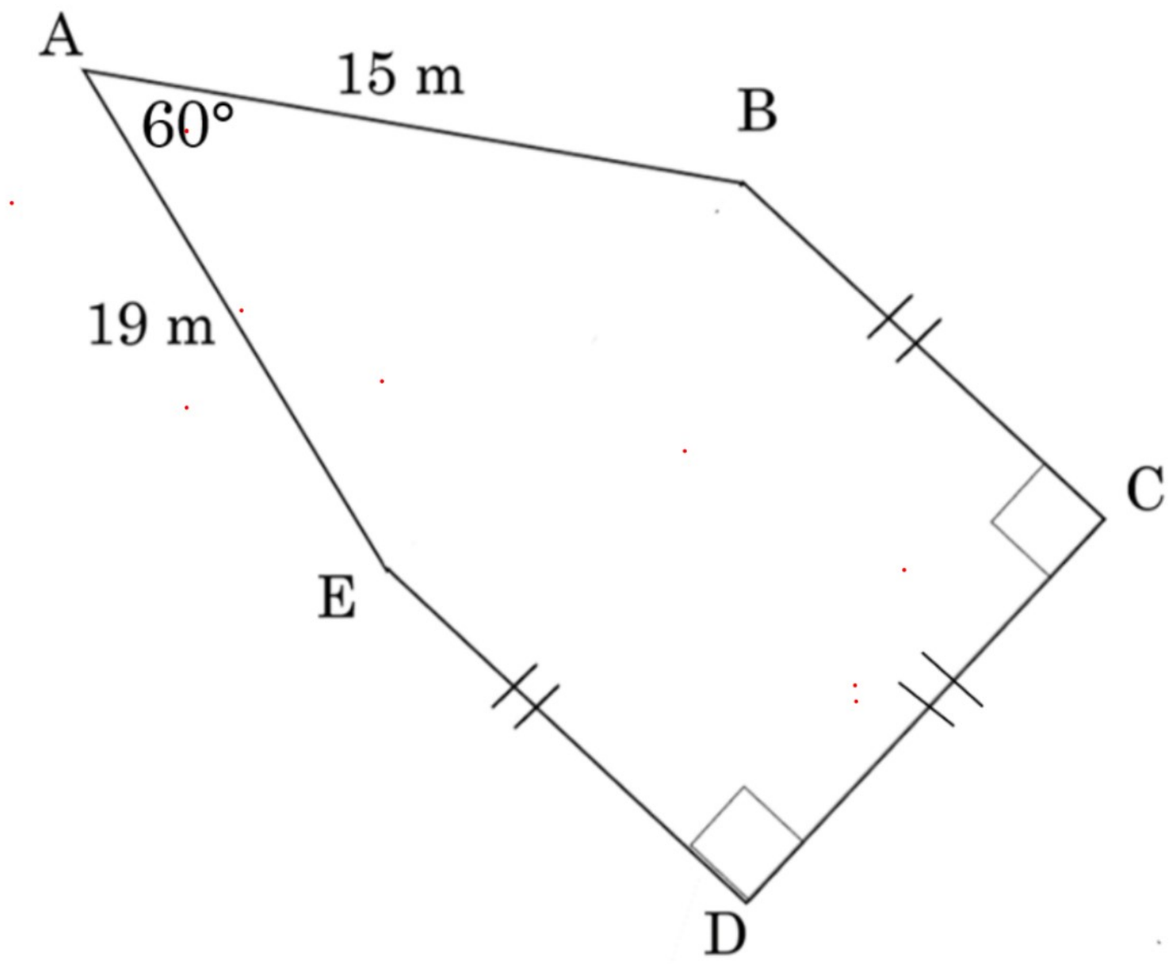
$$= \frac{1}{2}(16)(10)\frac{\sqrt{3}}{2}$$

$$= 40\sqrt{3}$$

Answer: $40\sqrt{3}$

(7 marks)

Q7. Below is a diagram of a garden:



Fencing is to be placed around the perimeter of the garden. If the fencing costs £7.50 per metre, work out the total cost.

$(BE)^2 = 15^2 + 19^2 - 2(15)(19)\cos(60)$

$(BE)^2 = 301$

$BE = \sqrt{301}$

perimeter $= 3\sqrt{301} + 15 + 19$

$= 86.048...$

cost $= 86.048 \times 7.50$

$= 645.360...$

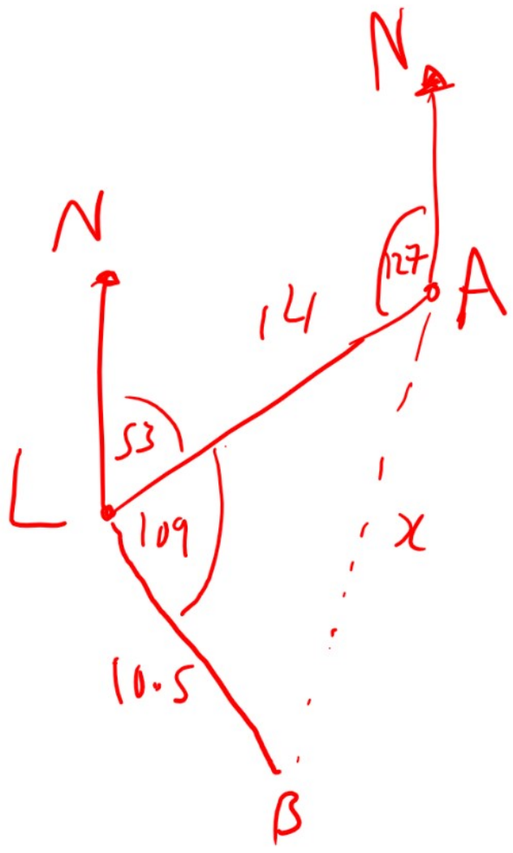
$= £645.36$

Answer: £645.36

(6 marks)

Q8. Ship A is 14 km from a lighthouse on a bearing of 053°. Ship B is 10.5 km from the same lighthouse on a bearing of 162°.

(i) Calculate the distance between the two ships, giving your answer to the nearest metre.



$x^2 = 14^2 + 10.5^2 - 2(14)(10.5)\cos(109)$

$x^2 = 401.967...$

$x = 20.049...$

$x = 20\text{km } 49\text{m}$

Answer: 20 km 49 m

(4 marks)

(ii) Calculate the bearing of ship B from ship A giving your answer to the nearest tenth of a degree.

- Required bearing $= 360^\circ - 127^\circ - \text{Angle } A$

- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{14^2 + 20.049^2 - 10.5^2}{2(14)(20.049)}$

$A = 29.682$

$\Rightarrow$ Bearing $= 203.3^\circ$

Answer: 203.3°

(3 marks)