Surds Exam Practice

Q1. Simplify the following expression showing all your steps:

$\sqrt{200} - \sqrt{32}$

$\sqrt{100}\sqrt{2} - \sqrt{16}\sqrt{2}$

$10\sqrt{2} - 4\sqrt{2}$

$6\sqrt{2}$

Answer: $6\sqrt{2}$

(2 marks)

Q2. Write $\sqrt{75}$ in the form $k\sqrt{3}$ where k is an integer to be found. You must show your working out.

$\sqrt{75} = \sqrt{25}\sqrt{3}$

$= 5\sqrt{3}$

$(k=3)$

Answer: $5\sqrt{3}$

(2 marks)

Q2. Simplify the following expression, showing all your working out:

$$\frac{3\sqrt{7}-5\sqrt{7}-6\sqrt{28}}{7}$$

$$= \frac{-2\sqrt{7} - 6\sqrt{4}\sqrt{7}}{7}$$

$$= \frac{-2\sqrt{7} - 12\sqrt{7}}{7}$$

$$= \frac{-14\sqrt{7}}{7}$$

Answer: $-2\sqrt{7}$

(3 marks)

Q4. Rationalise the denominator of each of the following expressions, and give your answer in its most simplified form:

a) $\frac{27}{3\sqrt{13}}$

$$\frac{27}{3\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{27\sqrt{13}}{39}$$

$$= \frac{9\sqrt{13}}{13}$$

Answer: $\frac{9\sqrt{13}}{13}$

(2 marks)

b) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{6-\sqrt{6}\sqrt{5}-\sqrt{6}\sqrt{5}+5}{6-5}$

$$= \frac{6-2\sqrt{30}+5}{1}$$

$$= 11-2\sqrt{30}$$

Answer: $11-2\sqrt{30}$

(3 marks)

Q5. Express $\sqrt{32} - \frac{14}{\sqrt{2}}$ in the form $k\sqrt{2}$ where k is an integer.

$$\sqrt{16}\sqrt{2} - \frac{14}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$4\sqrt{2} - \frac{14\sqrt{2}}{2}$$

$$4\sqrt{2} - 7\sqrt{2}$$

$$-3\sqrt{2} \quad (k = -3)$$

Answer: $-3\sqrt{2}$

(2 marks)

Q6. Show that the following expression: $\dfrac{1}{\frac{3}{\sqrt{2}} + \sqrt{2}} - \dfrac{\sqrt{2}}{5}$ is equal to 0.

$$\cdot \quad \frac{3}{\sqrt{2}} + \sqrt{2} = \frac{3 + \sqrt{2}\sqrt{2}}{\sqrt{2}}$$

$$= \frac{5}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{1}{\frac{5}{\sqrt{2}}} = \frac{\sqrt{2}}{5}$$

So expression becomes $\frac{\sqrt{2}}{5} - \frac{\sqrt{2}}{5} = 0$

Answer:

(3 marks)

Q7. Rationalise the denominator and simplify the following expression:
$\frac{b+\sqrt{c}}{b-\sqrt{c}}$ where b and c are positive integers.

$$\frac{b+\sqrt{c}}{b-\sqrt{c}} \times \frac{b+\sqrt{c}}{b+\sqrt{c}} = \frac{b^2+b\sqrt{c}+b\sqrt{c}+c}{b^2-b\sqrt{c}+b\sqrt{c}-c}$$

$$= \frac{b^2+2b\sqrt{c}+c}{b^2-c}$$

Answer: $\frac{b^2+2b\sqrt{c}+c}{b^2-c}$

(4 marks)

Q8. Let a be a positive integer. Simplify the following expression:
$\sqrt{a}\sqrt{a}-(\sqrt{a}-2a)^2$

$a-(\sqrt{a}-2a)(\sqrt{a}-2a)$

$a-\left[a-2a\sqrt{a}-2a\sqrt{a}+4a^2\right]$

$a-\left[a-4a\sqrt{a}+4a^2\right]$

$4a\sqrt{a}-4a^2$

$(=4a(\sqrt{a}-a))$

Answer: $4a\sqrt{a}-4a^2$

(2 marks)

Q9. Let c and d be any positive integers. Express $c\sqrt{d}$ in the form $\sqrt{E}$, where E is an expression in terms of c and d.

$$c\sqrt{d} = \sqrt{c^2}\sqrt{d}$$
$$= \sqrt{c^2 d}$$

$$E = \sqrt{c^2 d}$$

Answer: $E = \sqrt{c^2 d}$

(1 mark)

Applied Mixed Practice Problems

Q10. One cube has side length $3\sqrt{10}$, whilst another cube has side length $4\sqrt{5}$.

Work out the ratio of the volume of the smaller cube to the larger cube, giving your answer in the form n : m$\sqrt{2}$, where n, m are integers.

- Length scale factor ratio: $3\sqrt{10} : 4\sqrt{5}$

$\Rightarrow \sqrt{3^2\times 10} : \sqrt{4^2\times 5}$

$\Rightarrow \sqrt{90} : \sqrt{80}$

So smallest to largest is $\sqrt{80} : \sqrt{90}$

- volume scale factor ratio: $(\sqrt{80})^3 : (\sqrt{90})^3$

$\Rightarrow \sqrt{80}\sqrt{80}\sqrt{80} : \sqrt{90}\sqrt{90}\sqrt{90}$

$80\sqrt{80} : 90\sqrt{90}$

$8\sqrt{80} : 9\sqrt{90}$

$8\sqrt{16}\sqrt{5} : 9\sqrt{9}\sqrt{10}$

$8\times 4\sqrt{5} : 9\times 3\sqrt{10}$

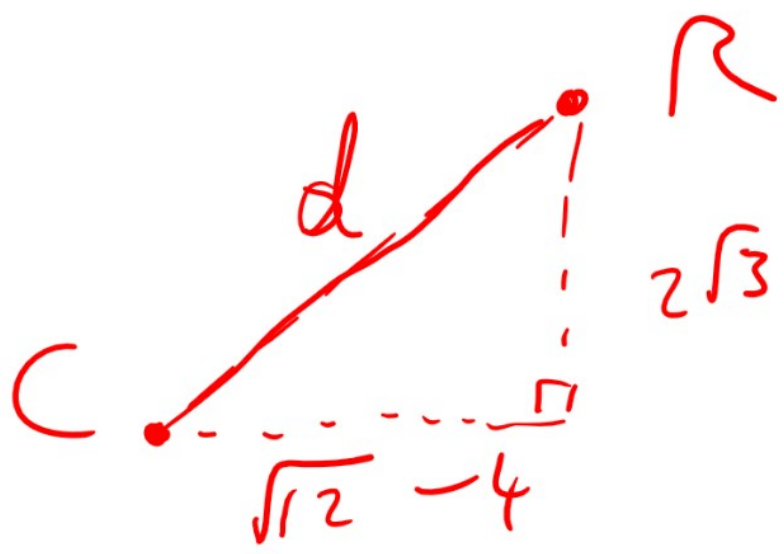
$32\sqrt{5} : 27\sqrt{10}$

$32\sqrt{5} : 27\sqrt{2}\sqrt{5}$

Answer: $32 : 27\sqrt{2}$

(4 marks)

Q11. A cathedral is $2\sqrt{3}$ miles due South and $\sqrt{12} - 4$ due West of a railway station. Show that the exact distance of the railway station to the cathedral can be expressed in the form $\sqrt{a(b - c\sqrt{3}}$, where a, b and c are integers.



$d^2 = (2\sqrt{3})^2 + (\sqrt{12} - 4)^2$ by Pythagoras.

$d^2 = 12 + 12 - 8\sqrt{12} + 16$

$d^2 = 40 - 8\sqrt{12}$

$d^2 = 40 - 8\sqrt{4}\sqrt{3}$

$d^2 = 40 - 16\sqrt{3}$

$d = \sqrt{40 - 16\sqrt{3}}$

$d = \sqrt{8(5 - 2\sqrt{3})}$

Answer: $d = \sqrt{8(5-2\sqrt{3})}$

(4 marks)

Q12. (i) Let a and b are positive integers where $a^2 > b^2$. State which of these statements must be true:

A) a > b B) a < b

eg test with a few values:
$3^2 > 2^2$ and $3 > 2$
$(0.5)^2 > (0.1)^2$ and $0.5 > 0.1$

Answer: a > b (A)

(1 mark)

(ii) Hence or otherwise, prove, using algebra, that $\sqrt{3} + \sqrt{7} > \sqrt{10}$.

LHS: $(\sqrt{3} + \sqrt{7})^2 = 3 + 2\sqrt{3}\sqrt{7} + 7$

$= 10 + 2\sqrt{3}\sqrt{7}$

RHS: $(\sqrt{10})^2 = 10$

As $2\sqrt{3}\sqrt{7} > 0$, $(\sqrt{3} + \sqrt{7})^2 > (\sqrt{10})^2$

By part (i), $\sqrt{3} + \sqrt{7} > \sqrt{10}$

Answer:________________

(3 marks)

Q13. Show that $3^{\frac{3}{2}} - 27(3^{-\frac{1}{2}})$ can be written in the form $k\sqrt{3}$ for some integer k.

$(\sqrt{3})^3 - \frac{27}{\sqrt{3}}$

$= 3\sqrt{3} - \frac{27\sqrt{3}}{3}$

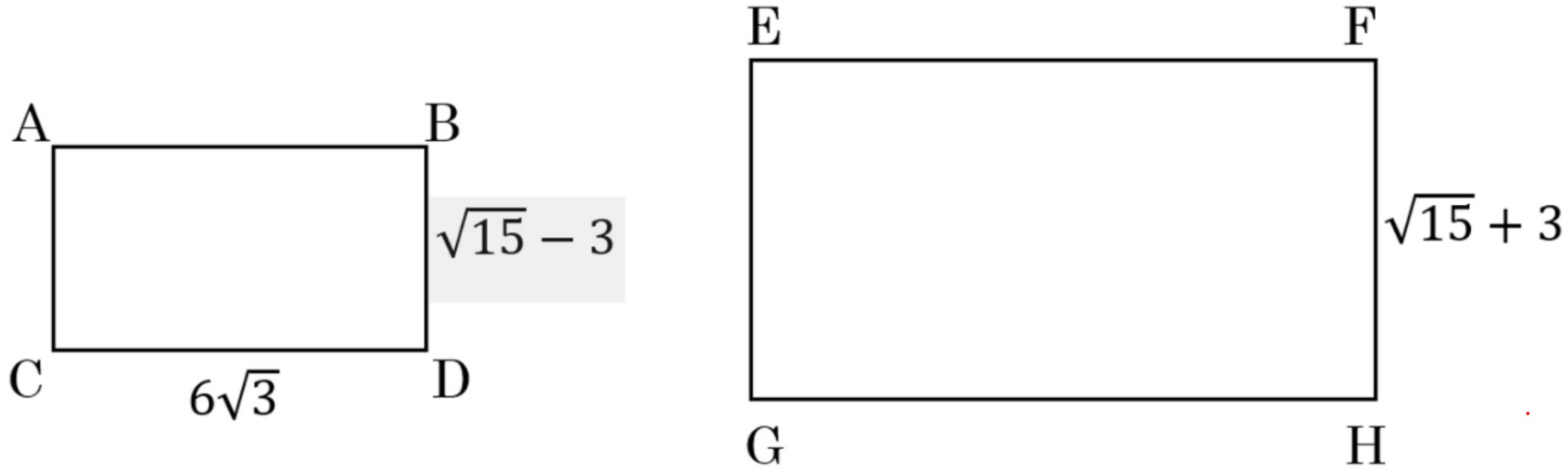
$= 3\sqrt{3} - 9\sqrt{3}$

$= -6\sqrt{3}$

Answer: $-6\sqrt{3}$

(3 marks)

Q14. ABCD and EFGH are similar shapes. Find the length of side GH, giving your answer in the form $a + \sqrt{b}$.



- Scale factor is $\dfrac{\sqrt{15}+3}{\sqrt{15}-3}$

$\Rightarrow \dfrac{\sqrt{15}+3}{\sqrt{15}-3} \times \dfrac{\sqrt{15}+3}{\sqrt{15}+3} = \dfrac{15+6\sqrt{15}+9}{15-9}$

$\Rightarrow \dfrac{24+6\sqrt{15}}{6} = 4+\sqrt{15}$

- GH is $6\sqrt{3} \times (4+\sqrt{15})$

$= 24\sqrt{3} + 6\sqrt{3}\sqrt{15}$

$= \sqrt{3}(24+6\sqrt{15})$

Answer: $\sqrt{3}(24+6\sqrt{15})$

(4 marks)