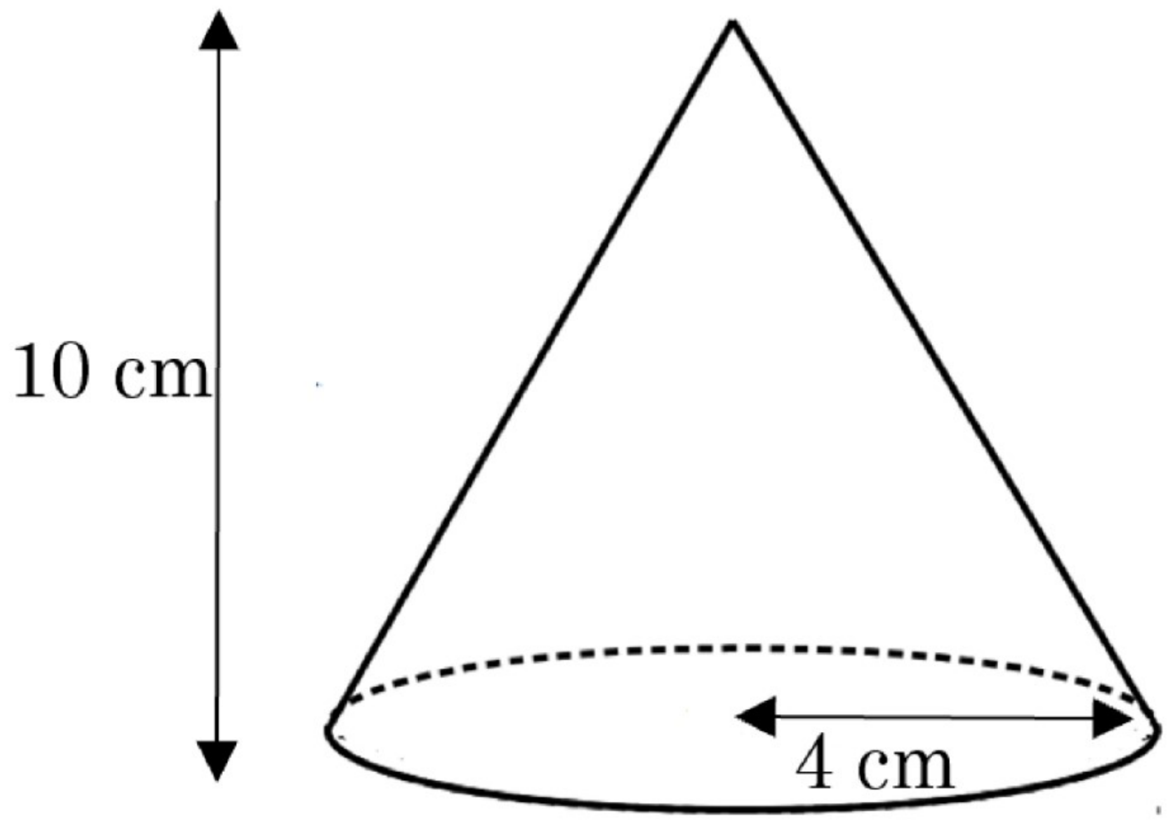
Spheres and Cones Exam Practice

Q1. The height of a cone is 10 cm and the radius of the base is 4 cm.
Work out the volume of the cone to 1 d.p.



$V = \frac{1}{3}\pi r^2 h$

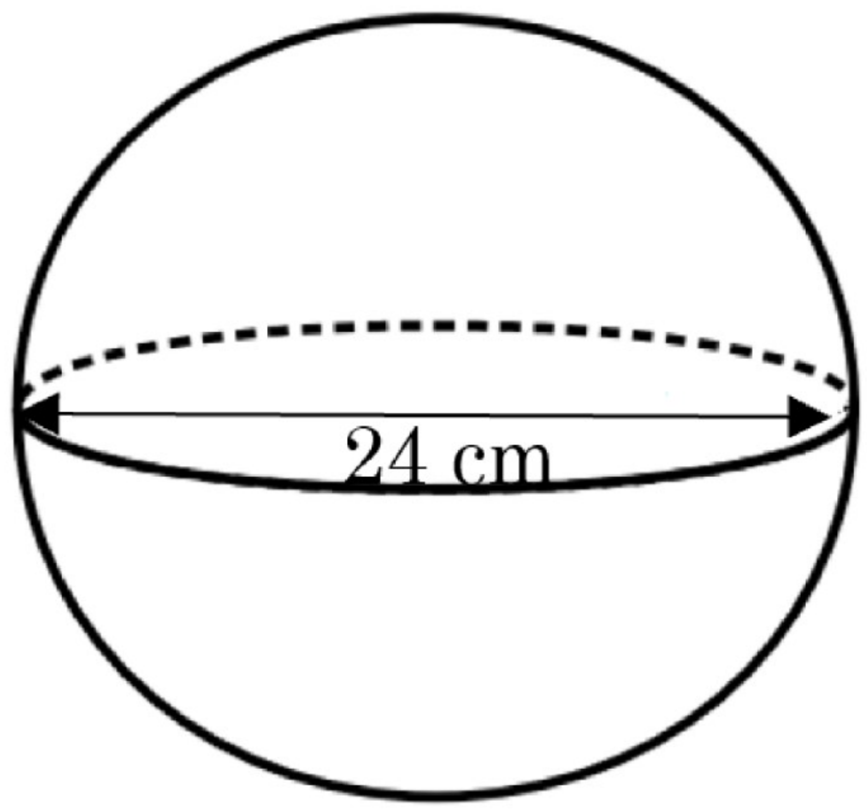
$= \frac{1}{3} \times \pi \times 4^2 \times 10$

$= 167.551...$

Answer: $167.6\ cm^3$

(2 marks)

Q2. A sphere has diameter 24cm. Find the volume of the sphere to 2 d.p.



$V = \frac{4}{3}\pi r^3$

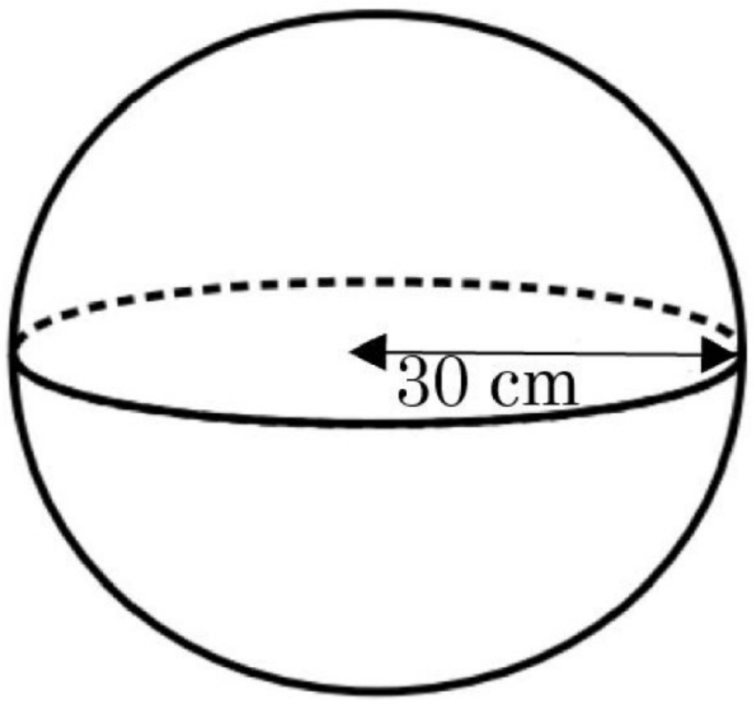
$= \frac{4}{3}\pi (12^3)$

$= 7238.229...$

Answer: $7238.23\ cm^3$

(2 marks)

Q3. Work out the surface area of the sphere with radius shown. Leave your answer in terms of π.



$S.A = 4\pi r^2$

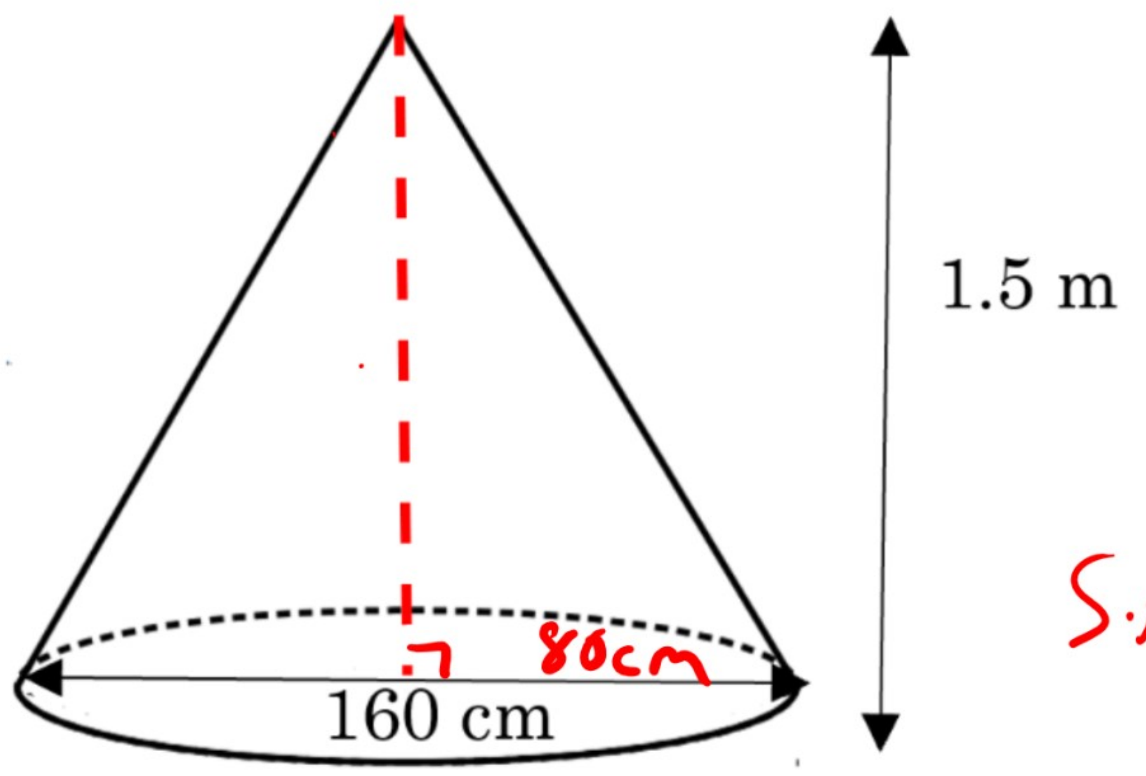
$= 4 \times \pi \times 15^2$

$= 900\pi$

Answer: 900π

(2 marks)

Q4. Work out the surface area of the cone shown. Leave your answer in terms of π.



$S.A = $ Curved $S.A.$ + base

$= \pi r l + \pi r^2$

$l = \sqrt{80^2 + 150^2}$ by Pythagoras' Theorem

$l = 170\text{cm}$

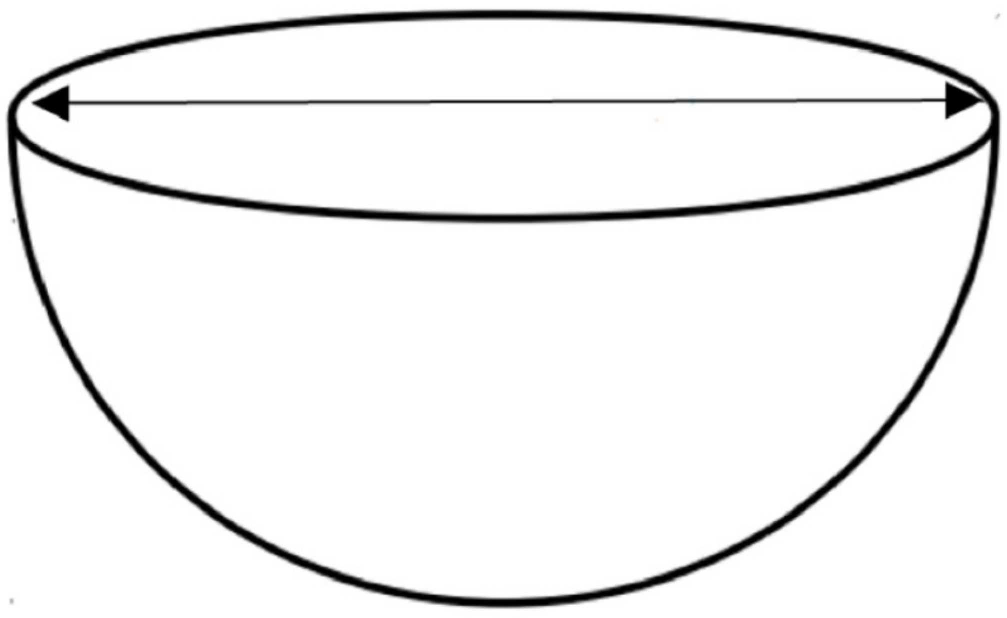
$\Rightarrow S.A. = 80\pi(170) + \pi(80)^2$

$= 13600\pi + 6400\pi$

Answer: $20000\pi \text{ cm}^3$

(2 marks)

Q5. The volume of the semi-sphere below is $\frac{2197\pi}{6}\text{cm}^3$. Find the diameter of the shape, shown.



$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$\frac{2197\pi}{6} = \frac{4}{6}\pi r^3$

$\Rightarrow 2197\pi = 4\pi r^3$

$\Rightarrow 2197 = 4r^3$

$\Rightarrow 549.25 = r^3$

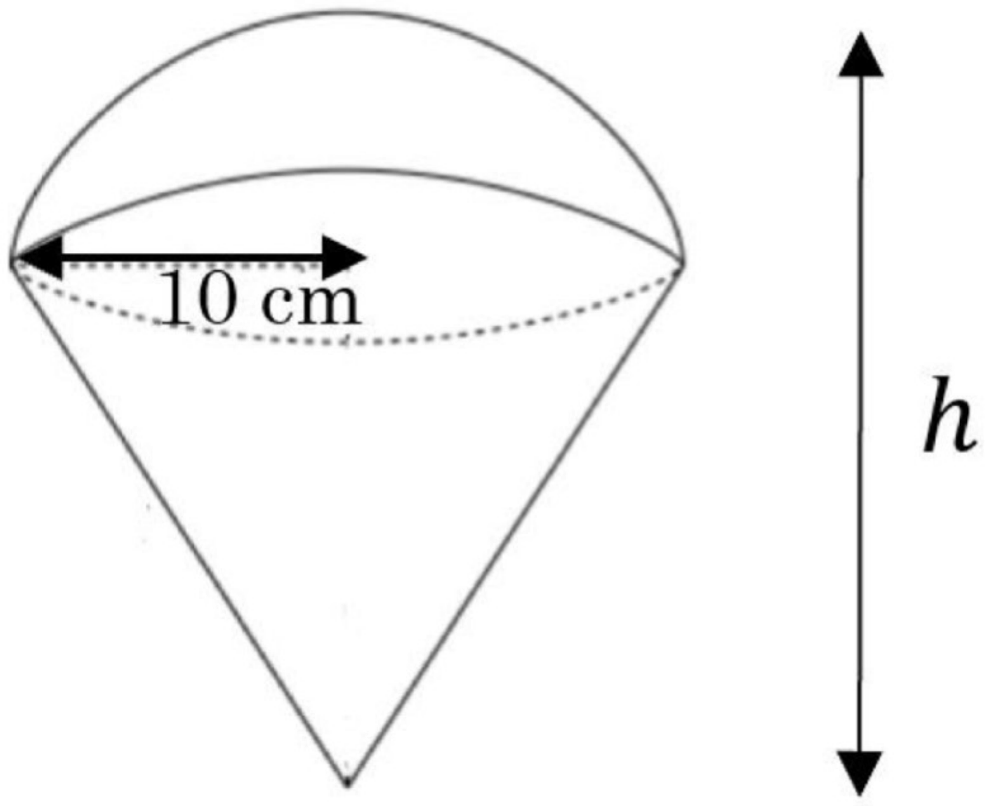
$\Rightarrow \sqrt[3]{549.25} = r$

$r = 8.189\ldots$

Answer: 8.2 cm

(3 marks)

Q6. The shape below is a cone with a hemi-sphere on top. If the volume of the shape is 2000π cm^3, find the height of the shape h.



$V = \frac{1}{3}\pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$2000\pi = \frac{1}{3}\pi(10^2)h + \frac{4}{6}\pi(10^3)$

$2000\pi = \frac{100\pi h}{3} + \frac{4000\pi}{6}$

$2000 = \frac{100h}{3} + \frac{4000}{6}$

$12000 = 200h + 4000$

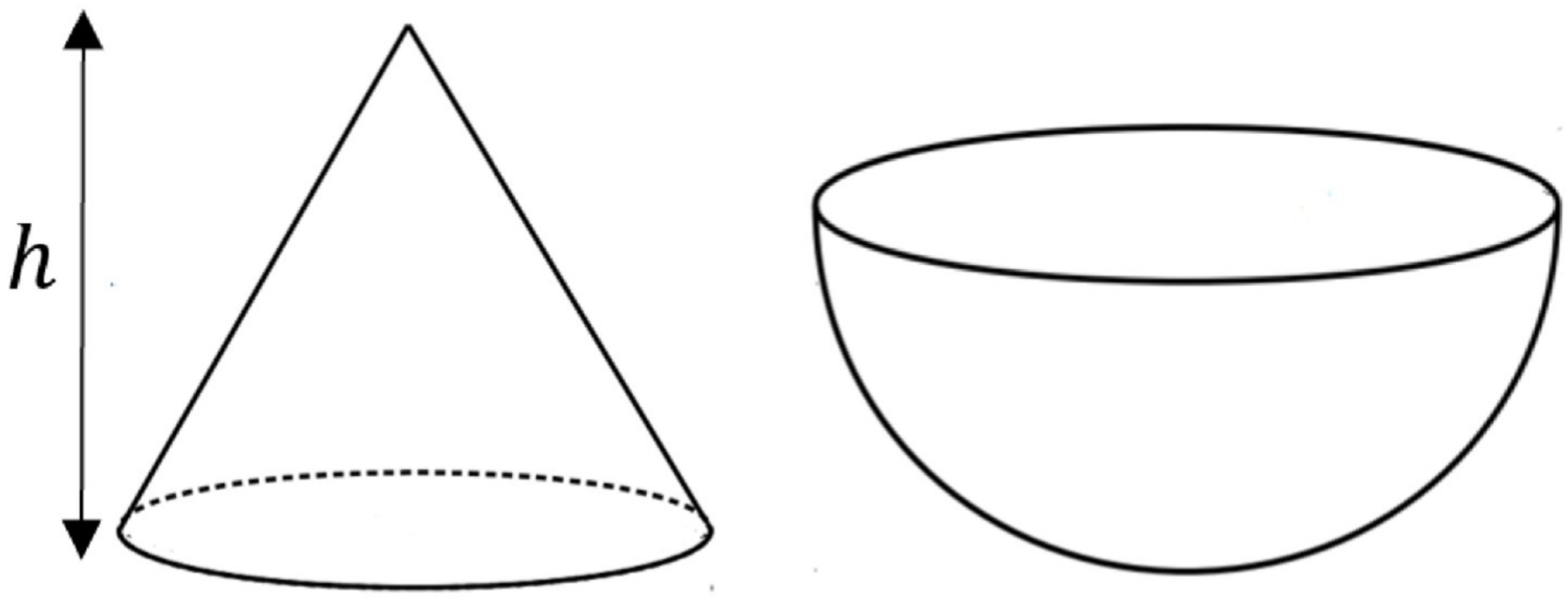
$8000 = 200h$

$\Rightarrow h = 40$

Answer: 40 cm

(3 marks)

Q7. The two shapes below have the same volume. The ratio of the radius of the cone to the diameter of the hemi-sphere is 2 : 5. Find an exact expression for the value of h.



- radius of cone : radius of hemi-sphere is 2 : 2.5
- let radius of the cone be 2r, radius of hemisphere be 10r

$$\frac{1}{3}\pi(2r)^2h = \frac{1}{2}\times\frac{4}{3}\pi\times(2.5^3)$$

$$\frac{4\pi r^2 h}{3} = \frac{125\pi r^3}{12}$$

$$\frac{4\pi h}{3} = \frac{125\pi r}{12}$$

$$\frac{4h}{3} = \frac{125r}{12}$$

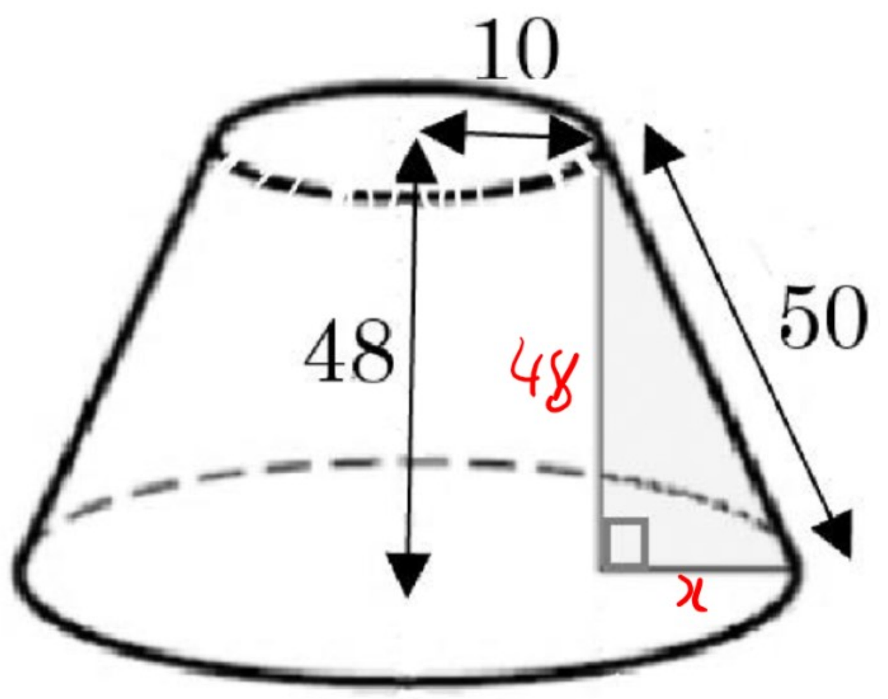
$$48h = 375r$$

$$h = \frac{375r}{48}$$

Answer: $h = \frac{125}{16}r$

(4 marks)

Q8. The top of the frustum below has a radius of 10 cm, and the perpendicular height of the shape is 48 cm. Find the volume to 3 s.f.



Volume = large cone − small cone

$x = \sqrt{50^2 - 48^2}$ (Pythagoras)

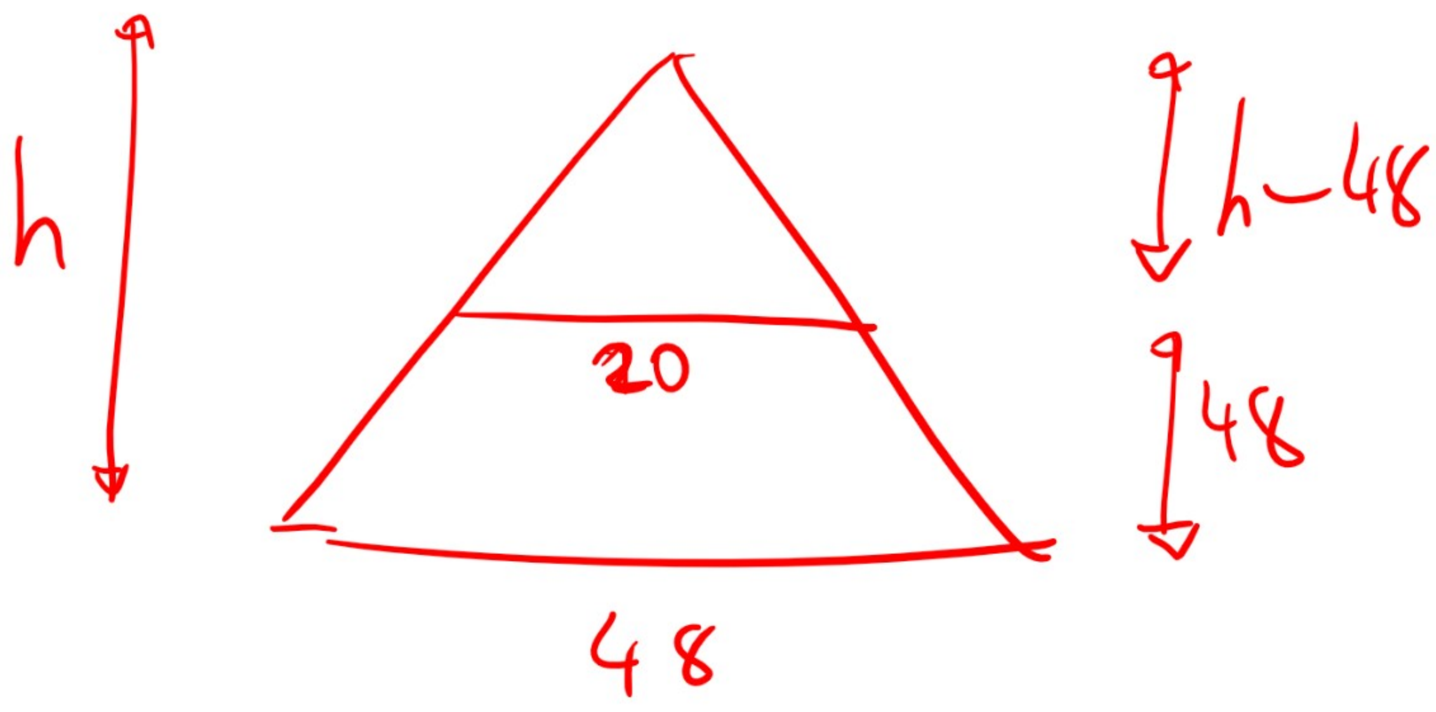
$= \sqrt{196}$

$= 14$

⇒ radius base of frustum $= 14 + 10$

$= 24$

⇒ next find height of large cone by similar triangles



$$\frac{48}{20} = \frac{h}{h-48}$$

$48h - 2304 = 20h$

$28h = 2304$

$h = 82.286$

Vol. small cone : $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(10^2)(82.286 - 48)$

$= 3590.421\ldots$

Vol. large cone: $\frac{1}{3}\pi(24^2)(82.286)$

$= 49633.745\ldots$

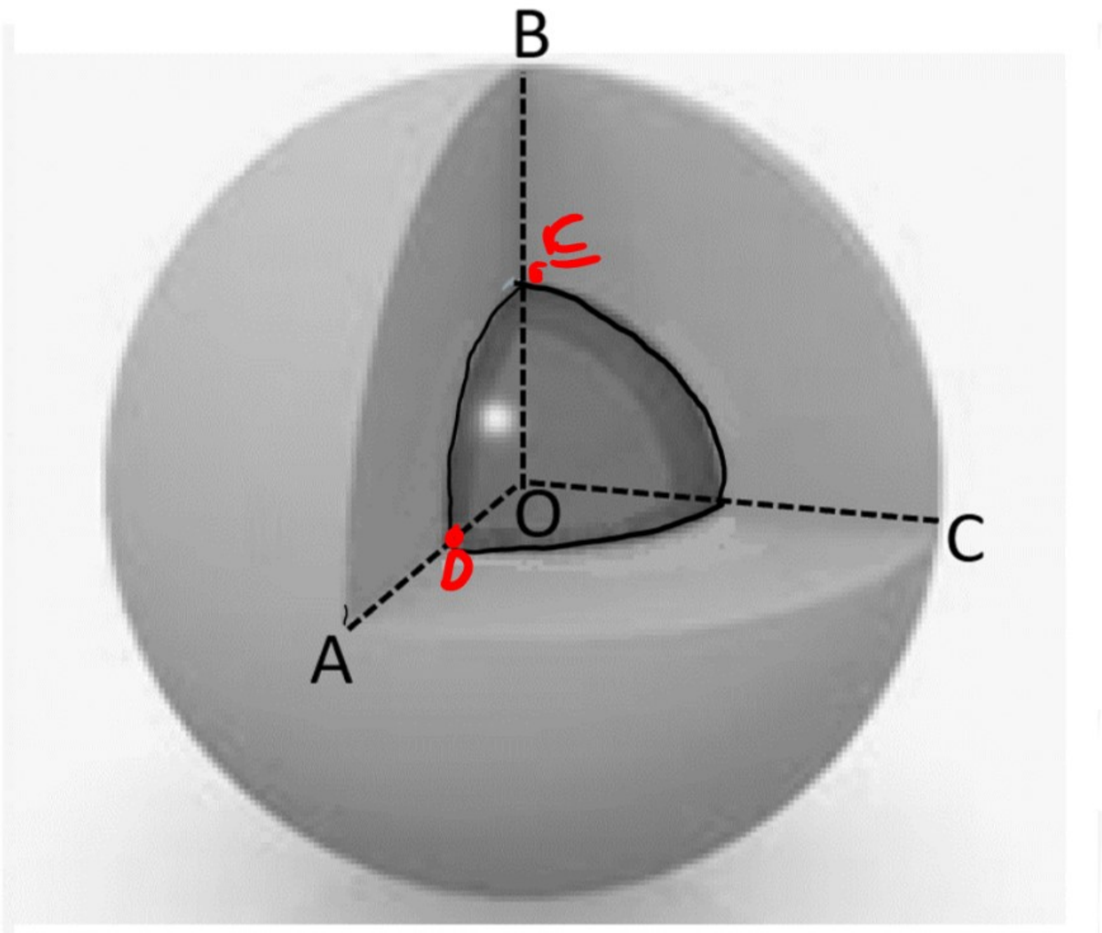
Vol frustum $= 46043.32\ldots\ldots$

Answer: 46000 cm^3

(3 marks)

Q9. A model of the planet Mercury is made in the shape below, consisting of a smaller sphere, representing the planets core, inside a larger sphere. As shown, a portion of the shape has been removed to reveal the core, from which nothing has been removed.

The centre of the model is O, with angles AOB and BOC are 90°. If the radius of the larger sphere is 15 cm and the radius of the smaller sphere is 9 cm, find the surface area of the model.



Total S.A = 'Outer' surface + Inner surface (from removed section)

• Outer surface $= \frac{7}{8} \times 4\pi r^2$

$= \frac{7}{8} \times 4\pi(15^2)$

$= \frac{1575\pi}{8}$ cm^2

• Inner surface = S.A. of core visible + 3× area of annulus sector ABDE

• S.A. of core $= \frac{1}{8} \times 4\pi(9^2)$ (using $\frac{1}{8} \times 4\pi r^2$)

• Annulus sector = Sector OAB − Sector ODE

$= \frac{1}{4} \times \pi \times 15^2 - \frac{1}{4}\pi \times 6^2$

$= \frac{189\pi}{4}$

⇒ Inner surface $= \frac{1}{8} \times 4\pi(81) + 3 \times \frac{189}{4}\pi$

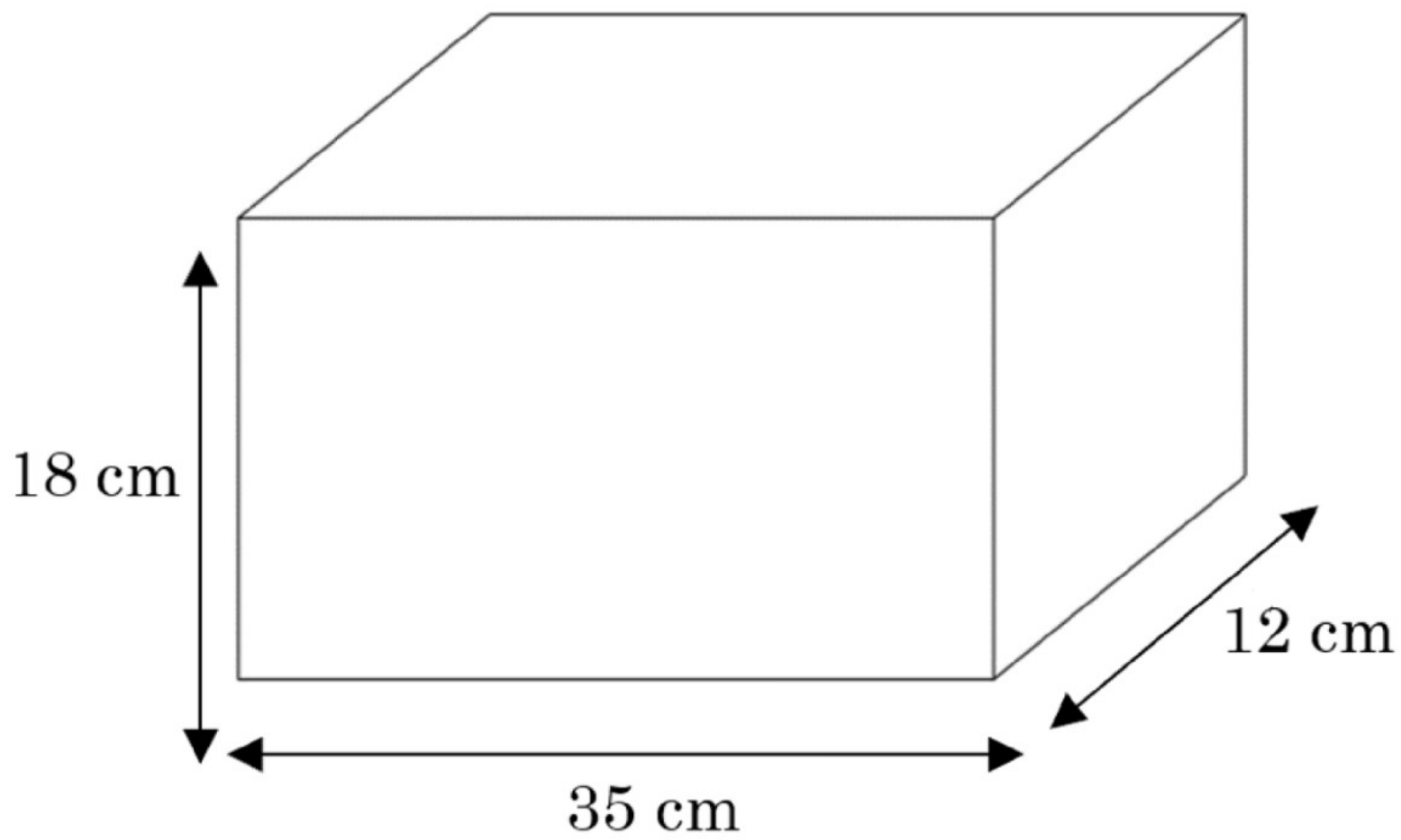
$= \frac{729}{4}\pi$

∴ Total S.A $= \frac{1575\pi}{8} + \frac{729\pi}{4}$

Answer: $\frac{3033\pi}{8}$ (1191.1 cm^2)

(4 marks)

Q10. Below is a tank in the shape of a cuboid. As part of an experiment, it is filled to two-thirds of its capacity with water.
A number metal spheres, each of radius 8 mm, are dropped into the water and sink to the bottom. Work out the number of spheres required to raise the water level in the tank by at least 5%.



- original height of water $= \frac{2}{3} \times 18$
  $= 12$ cm
- If raises by 5%, height of water will be $1.05 \times 12 = 12.6$ cm
- original volume of water $= 12 \times 35 \times 12$ (h)
  $= 5040$
- new vol. of water $= 35 \times 12 \times 12.6$
  $= 5292$

$\Rightarrow$ need increase of 252 cm^3

- Each sphere has vol. $\frac{4}{3} \times \pi \times 0.8^3 = 2.144...$

$252 \div 2.144... = 117.5...$

Answer: 118 spheres

(3 marks)