

Solving Equations Using Iteration Exam Practice



- Q1. Working to 3 d.p. use the following iteration formula $x_{n+1} = \sqrt[3]{\frac{3x_n + 5}{2}}$, starting with $x_0 = 2$, to find the values of x_1 , x_2 , x_3 and x_4 .
(3 marks)
- Q2. Working to 3 d.p. use the following iteration formula $x_{n+1} = \frac{4x_n^2 + 6}{3}$, starting with $x_0 = 1$, to find the values of x_1 , x_2 , x_3 and x_4 .
(3 marks)
- Q3. Using $x_{n+1} = 9 - \frac{5}{x_n^2}$, with $x_0 = 1$, find the values of x_1 , x_2 and x_3 to 2 d.p.
(3 marks)
- Q4. Using $x_{n+1} = 3.2 + \frac{15}{\sqrt{x_n}}$, with $x_0 = 1$, find the values of x_1 , x_3 and x_7 correct to 2 d.p.
(3 marks)
- Q5. Using $x_{n+1} = \frac{3}{5} + \frac{7x_n}{5}$, with $x_0 = 1.5$, state the first term of the sequence which is larger than 30, correct to 3 d.p.
(2 marks)
- Q6. A sequence is defined using the formula $x_{n+1} = \frac{3x_n^2 + 2}{4x_n^2 + 7}$ with $x_0 = 2.9$.
a) Find the values of x_1 , x_2 and x_3 correct to 4 decimal places.
(2 marks)
b) State the difference between x_{9999} and x_{10000} to 4 decimal places.
(1 mark)
- Q7. A sequence is defined $x_{n+1} = \frac{20}{4x_n + 6}$, together with a value for x_0 .
a) If the value of x_1 is work out the value of x_0 .
(2 marks)
b) Find the values of x_2 , x_3 and x_4
(2 marks)
- Q8. A sequence is defined $x_{n+1} = x_n^2 + 4$, together with a value for x_0 .
If the value of x_1 is 13, work out two possible values for x_0 .
(3 marks)



Applied Mixed Practice Problems

Q9.a) Show that the equation $2x^3 - 3x + 5 = 0$ has a solution in the interval $1.5 < x < 2.5$. (2 marks)

b) Show that the equation $2x^3 - 3x + 5 = 0$ can be written in the form

$$x = \sqrt[3]{\frac{3x+5}{2}} \quad (2 \text{ marks})$$

c) Solve the equation $2x^3 - 3x + 5 = 0$ correct to 4 decimal places, using the iteration,

$$x_0 = 2, \quad x_{n+1} = \sqrt[3]{\frac{3x_n+5}{2}} \quad (2 \text{ marks})$$

Q10. a) Starting with $x_0 = 0.2$, use the iteration formula $x_{n+1} = \sqrt{x_n + \frac{3}{4}}$ three times to find an estimate for the equation $4x^2 - 4x - 3 = 0$. (2 marks)

b) Find the exact solutions of the equation $4x^2 - 4x - 3 = 0$. (2 marks)

c) Calculate the percentage error of your answer to part (a) (2 marks)

Q11. The population P_d (in 100's) of a colony of beetles is modelled over time d (days) by the iteration formula:

$$P_0 = 20, \\ P_{d+1} = 1.04(P_d - 1) + 0.75$$

a) Use the model to work out how many beetles there will be after 4 days (2 marks)

b) After how many complete days will there be more than 2500 beetles? (2 marks)

Q12. a) Find two different iterative formulae of the form $x_{n+1} = f(x_n)$ to solve the equation, $x^2 - 5x - 3 = 0$. (2 marks)

b) Hence find both roots of the equation $x^2 - 5x - 3 = 0$. You must show all your working out. (2 marks)