Solving Equations Using Iteration Exam Practice

Q1. Use the following iteration formula $x_{n+1} = \sqrt[3]{\frac{3x_n + 5}{2}}$,

starting with $x_0 = 2$, to find the values of x_1, x_2, x_3 and x_4.

Give your answers to 3 d.p.

$x_1 = 1.7651\ldots$

$x_2 = 1.7266\ldots$

$x_3 = 1.7201\ldots$

$x_4 = 1.7190\ldots$

Answer: 1.765, 1.727, 1.720, 1.720

(3 marks)

Q2. Use the following iteration formula $x_{n+1} = \frac{4x_n{}^2 + 6}{3}$,

starting with $x_0 = 1$, to find the values of x_1, x_2, x_3 and x_4.

Give your answers to 3 d.p.

$x_1 = 3.3333\ldots$

$x_2 = 16.8148\ldots$

$x_3 = 378.9839\ldots$

$x_4 = 191507.1593\ldots$

Answer: 3.333, 16.815, 378.984, 191507.159 (3 marks)

Q3. Using $x_{n+1} = 9 - \frac{5}{{x_n}^2}$, with $x_0 = 1$, find the values of x_1, x_2 and x_3.

Give your answers to 3 d.p.

$x_1 = 4.000$

$x_2 = 8.6875$

$x_3 = 8.9337\ldots$

Answer: 4.000, 8.688, 8.934

(3 marks)

Q4. Using $x_{n+1} = 3.2 + \frac{15}{\sqrt{x_n}}$, with $x_0 = 1$, find the values of x_1, x_3 and x_7 correct to 2 d.p.

$x_1 = 18.20$

$x_2 = 6.716\ldots$

$x_3 = 8.988\ldots$

$x_4 = 8.203\ldots$

$x_5 = 8.437\ldots$

$x_6 = 8.364\ldots$

$x_7 = 8.386\ldots$

Answer: 18.20, 8.99, 8.39

(2 marks)

Q5. Using $x_{n+1} = \frac{3}{5} + \frac{7x_n}{5}$, with $x_0 = 1.5$, state the first term of the sequence which is larger than 30, correct to 3 d.p.

$x_1 = 2.7$

$\vdots$

$x_7 = 30.1240\ldots$

Answer: 30.124

(2 marks)

Q6. A sequence is defined using the formula $x_{n+1} = \frac{3x_n^2 + 2}{4x_n^2 + 7}$ with $x_0 = 2.9$.

a) Find the values of x_1, x_2 and x_3 correct to 4 decimal places.

$x_1 = 0.67002\ldots$

$x_2 = 0.38050\ldots$

$x_3 = 0.32119\ldots$

Answer: $0.6700, 0.3805, 0.3212$

(3 marks)

b) State the difference between x_{9999} and x_{1000} to 4 decimal places

Answer: 0

(1 mark)

Q7. A sequence is defined $x_{n+1} = \frac{20}{4x_n + 6}$, together with a value for x_0.

a) If the value of x_1 is 2.5, work out the value of x_0.

$$2.5 = \frac{20}{4x_0 + 6}$$

$$4x_0 + 6 = \frac{20}{2.5}$$

$$4x_0 = 2$$

$$x_0 = 0.5$$

Answer: $x_0 = 0.5$

(2 marks)

b) Find the values of x_2, x_3 and x_4 to 3 d.p.

$$x_2 = 1.250$$

$$x_3 = 1.8181\ldots$$

$$x_4 = 1.5068\ldots$$

Answer: 1.250, 1.818, 1.507

(2 marks)

Q8. A sequence is defined $x_{n+1} = x_n{}^2 + 4$, together with a value for x_0.

If the value of x_1 is 13, work out two possible values for x_0.

$$13 = x_0{}^2 + 4$$

$$9 = x_0{}^2$$

$$x_0 = \pm\sqrt{9}$$

$$x_0 = -3, 3$$

Answer: $x_0 = 3$ or -3

(2 marks)

Problem Questions:

Q9.a) Show that the equation $2x^3 - 3x + 5 = 0$ has a solution in the interval

$-2 < x < -1$

Let $p(x) = 2x^3 - 3x + 5$

$p(-2) = -5$

$p(-1) = 9$

Change in sign of $p(x)$ over the interval $\Rightarrow$ exists solution in the interval

Answer: _____

(2 marks)

b) Show that the equation $2x^3 - 3x + 5 = 0$ can be written in the form

$x = \sqrt[3]{\frac{3x-5}{2}}$

$2x^3 = 3x - 5$

$x^3 = \frac{3x-5}{2}$

$x = \sqrt[3]{\frac{3x-5}{2}}$

Answer: _____

(2 marks)

c) Solve the equation $2x^3 - 3x + 5 = 0$ correct to 4 decimal places, using the iteration,

$x_0 = -2, \quad x_{n+1} = \sqrt[3]{\frac{3x_n - 5}{2}}$

$x_1 = -1.76517\ldots$

$\vdots$

The sequence stabilises to : $-1.71885\ldots$

$\Rightarrow x = -1.7189$

Answer: -1.7189

(2 marks)

Q10. a) Starting with $x_0 = 0.2$, use the iteration formula $x_{n+1} = \sqrt{x_n + \frac{3}{4}}$ 4 times to find an estimate for the equation $4x^2 - 4x - 3 = 0$.

Give your answer correct to 2 decimal places.

$x_1 = 0.974\ldots$

$x_2 = 1.313\ldots$

$x_3 = 1.436\ldots$

$x_4 = 1.478\ldots$

Answer: 1.48

(2 marks)

b) Find the exact solutions of the equation $4x^2 - 4x - 3 = 0$.

$a = 4$, $b = -4$, $c = -3$

$$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - (4)(4)(-3)}}{2(-3)}$$

Answer: $x = 1.5, -0.5$

(2 marks)

c) Calculate the percentage error of your answer to part (a)

$$\% \text{ error} = \frac{\text{error}}{\text{actual}} \times 100$$

$$= \frac{1.5 - 1.48}{1.5} \times 100$$

$$= 4.3\%$$

Answer: 4.3%

(2 marks)

Q11. The population P_d (in 100's) of a colony of beetles is modelled over time d (days) by the iteration formula:

$P_0 = 20,$
$P_{d+1} = 1.04(P_d - 1) + 0.75$

a) Use the model to work out how many beetles there will be after 4 days

$P_1 = 1.04(20) + 0.75$
$= 21.55$

$P_2 = 1.04(21.55) + 0.75$
$= 22.122$

$P_3 = 22.716...$

$P_4 = 23.335...$

Answer: 23 beetles

(2 marks)

b) After how many complete days will there be more than 2500 beetles?

$P_5 = 23.978$
⋮
$P_{12} = 29.264...$
$P_{13} = 30.144...$

Answer: 13 days

(2 marks)

Q12. a) Find two different iterative formulae of the form $x_{n+1} = f(x_n)$ to solve the equation, $x^2 - 5x - 3 = 0$.

1) $x^2 = 5x + 3$

$x = 5 + \frac{3}{x}$

$\Rightarrow x_{n+1} = 5 + \frac{3}{x_n}$

2) $x(x-5) = 3$

$x = \frac{3}{x-5}$

$\Rightarrow x_{n+1} = \frac{3}{x_n - 5}$

Answer: _____

(2 marks)

b) Hence find both roots of the equation $x^2 - 5x - 3 = 0$ correct to 3 decimal places. You must show all your working out.

- $x_{n+1} = 5 + \frac{3}{x_n}$

Try $x_0 = 0.5$, for example

$\Rightarrow x = 5.541$

- $x_{n+1} = \frac{3}{x_n - 5}$

Try $x_0 = 0.5$, for example

$\Rightarrow x = -0.541$

Answer: 5.541, −0.541

(2 marks)