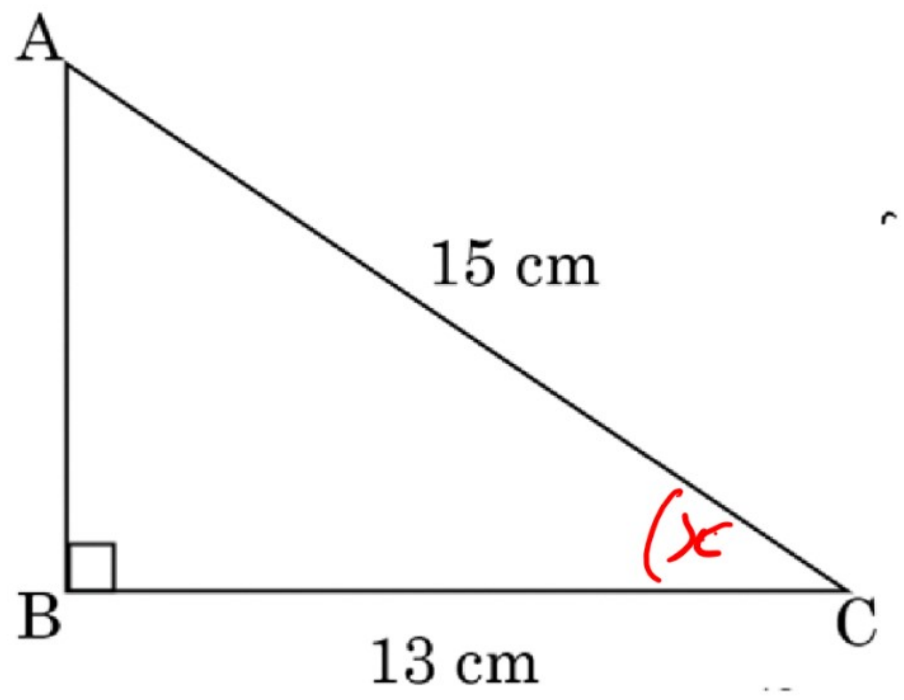
Trigonometry Exam Practice

Q1. Find the size of angle ACB to 1 decimal place.



$\cos(x) = \frac{13}{15}$

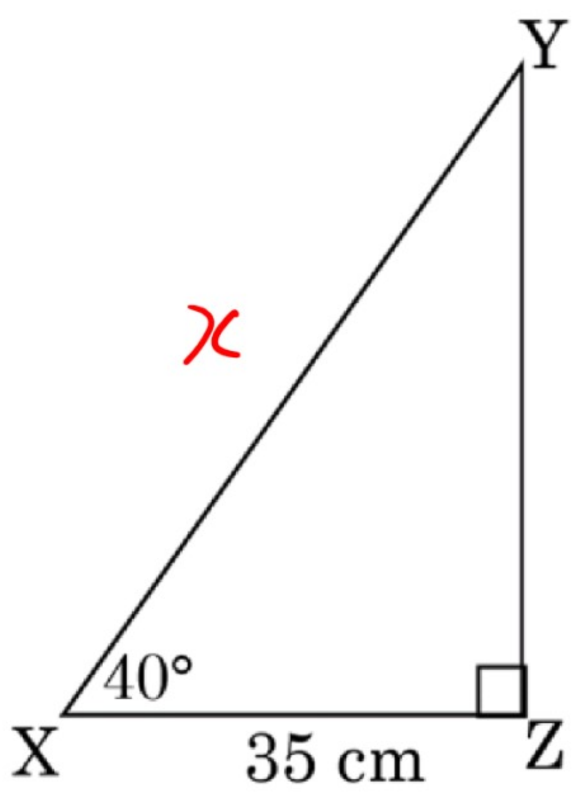
$x = \cos^{-1}\left(\frac{13}{15}\right)$

$x = 29.92\ldots$

Answer: 29.9°

(3 marks)

Q2. Find the length of side XY to 1 decimal place.



$\cos(40) = \frac{35}{x}$

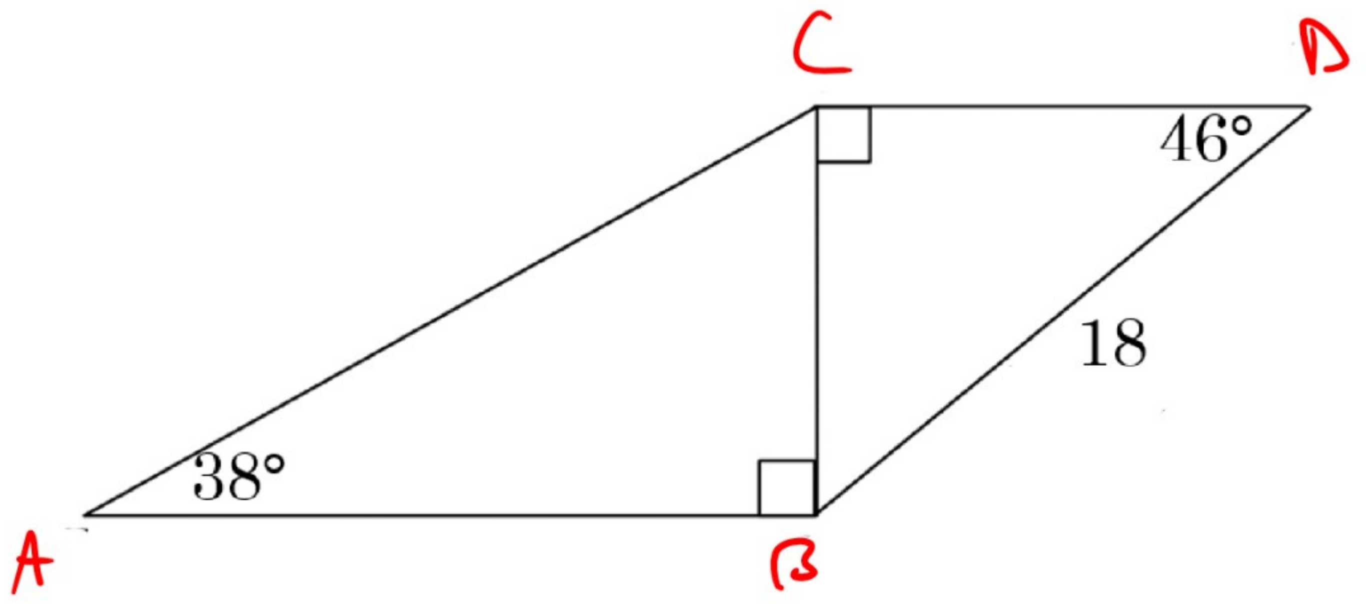
$x = \frac{35}{\cos(40)}$

$x = 49.49\ldots$

Answer: 49.5cm

(3 marks)

Q3. Find the perimeter of the shape below correct to 2 decimal places.



• $\cos(46) = \frac{CD}{18}$

$\Rightarrow CD = \underline{12.504}$ ..

• $\sin(46) = \frac{BC}{18} \Rightarrow BC = 12.948$.

• $\sin(38) = \frac{12.948}{AC} \Rightarrow AC = \underline{7.972}$

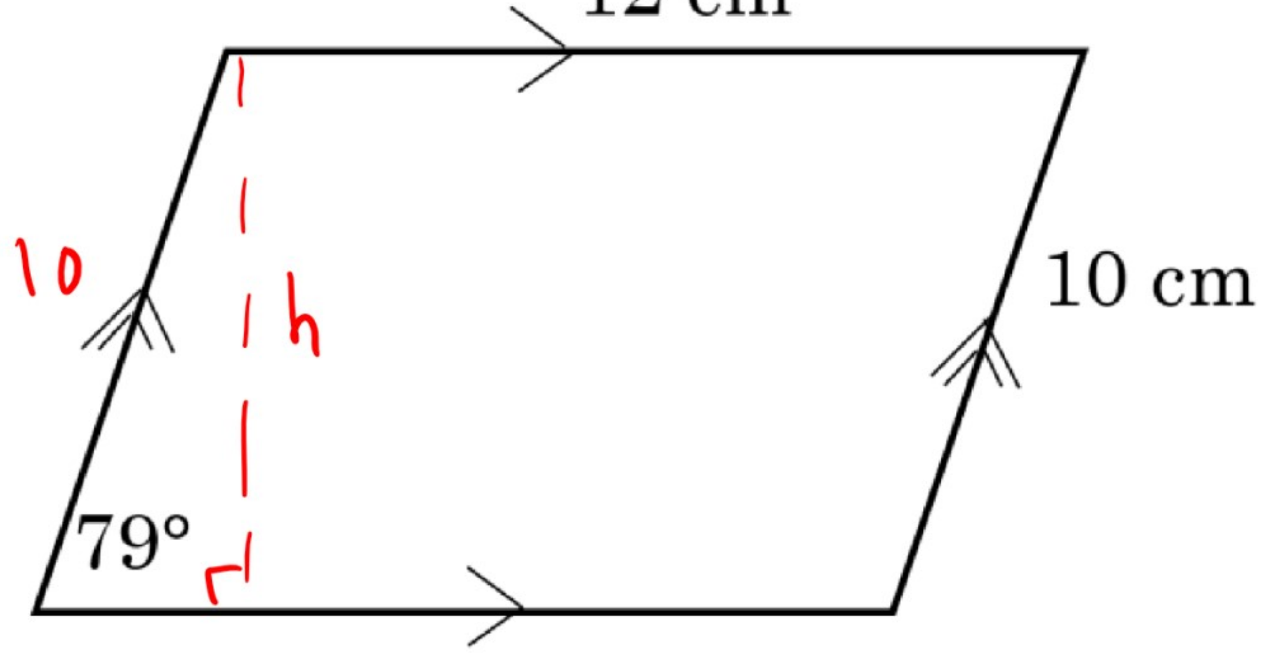
perimeter = $18 + CD + AC + AB$

• $\cos(38) = \frac{AB}{7.972} \Rightarrow AB = \underline{6.282}$

Answer: 44.76

(4 marks)

Q4. Find the area of the shape below correct to 1 decimal place.



• $\sin(79) = \frac{h}{10} \Rightarrow h = 10\sin(79)$

• Area = length × perpendicular height (parallelogram)

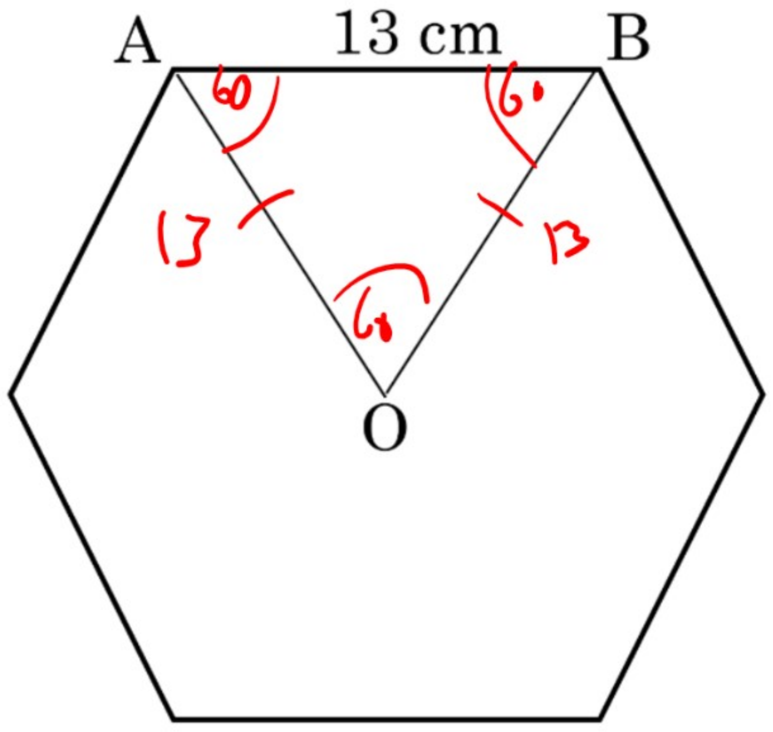
$= 10\sin(79) \times 12$

$= 117.795...$

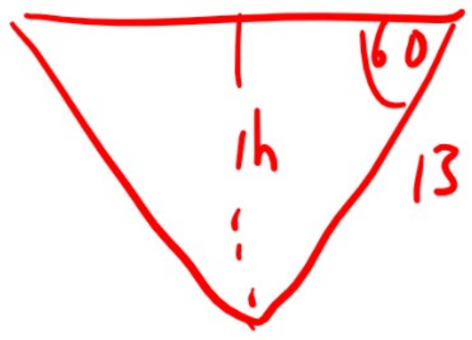
Answer: 117.8 units^2

(4 marks)

Q5. Below, O is the centre of the regular hexagon shown. Find the area of the hexagon to the nearest square cm.



• Area one triangle $= \frac{1}{2} \times b \times h$



$\sin(60) = \frac{h}{13} \Rightarrow h = 13\sin(60)$

Area $= \frac{1}{2} \times 13 \times 13\sin(60)$

• Area hexagon $= 6\left(\frac{1}{2} \times 13 \times 13 \times \sin 60\right)$

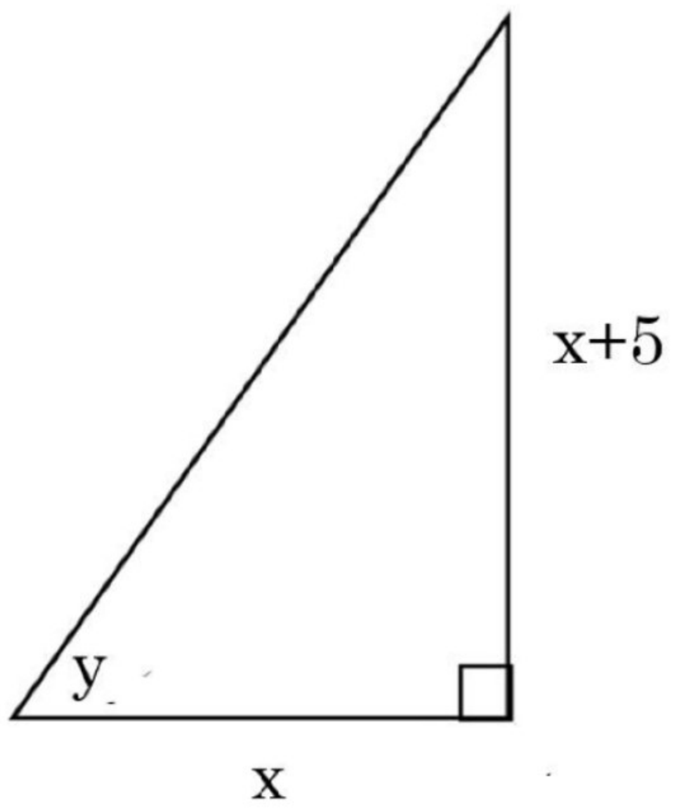
$= 507\sin(60)$

$= 439.07\ldots$

Answer: 439 cm^2

(4 marks)

Q6. The area of the triangle below is 18 cm². Find the size of angle y to 1 decimal place.



• $\frac{1}{2} \times x \times (x+5) = 18$

$x^2 + 5x = 36$

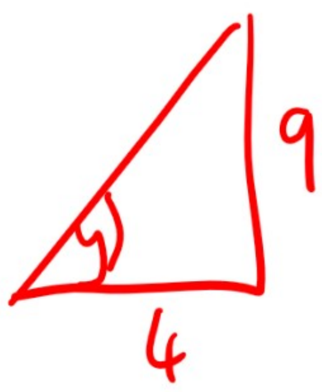
$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$x = -9, \; 4$ (✓)

reject

•



$\tan(y) = \frac{9}{4}$

$y = \tan^{-1}\left(\frac{9}{4}\right)$

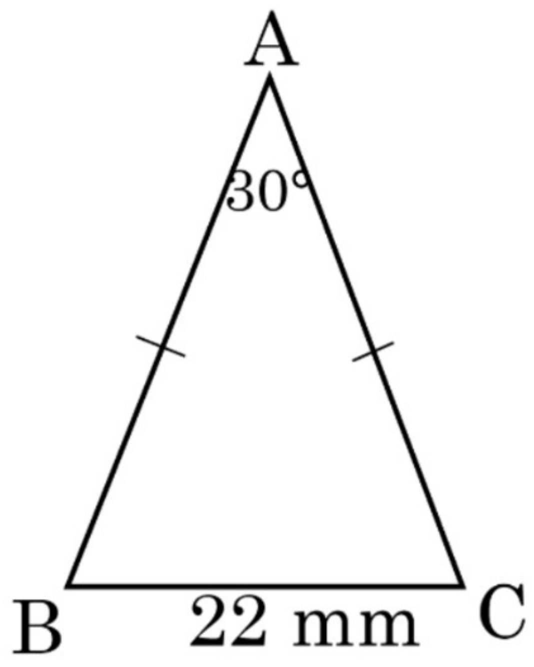
$y = 66.03\ldots$

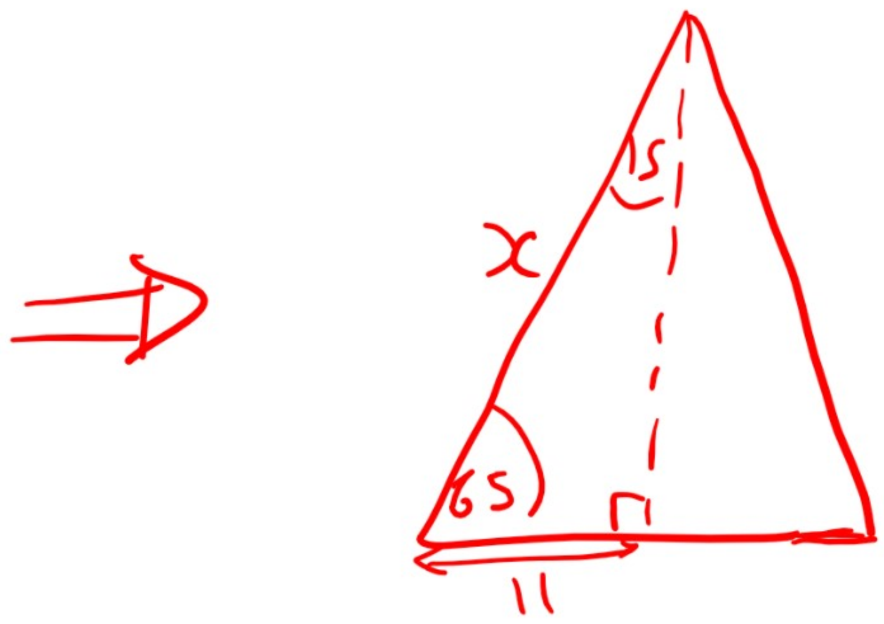
Answer: $66.0°$

(5 marks)

Problem Questions:

Q7. A design for the face of a watch consists of a number of metal strips bent into the shape of triangles as shown below:





The triangles are arranged in a circular pattern so that each vertex A meets at a point. Sufficient triangles are used so there is no gap left.

Estimate the total length of the metal used to 1 decimal place. Why is the model unrealistic?

- $\cos(65) = \frac{11}{x} \Rightarrow x = \frac{11}{\cos(65)}$

  $x = 26.028$

- $360 \div 30° \Rightarrow$ there will be 12 triangles in the design.

- Total of radial lines (AB's and AC's) $= 26.028 \times 12$

  $= 312.338$

- Total of lines of type BC $= 22 \times 11$

  $= 242$

- Total length of metal $= 312.338 + 242$

  $= 554.338$

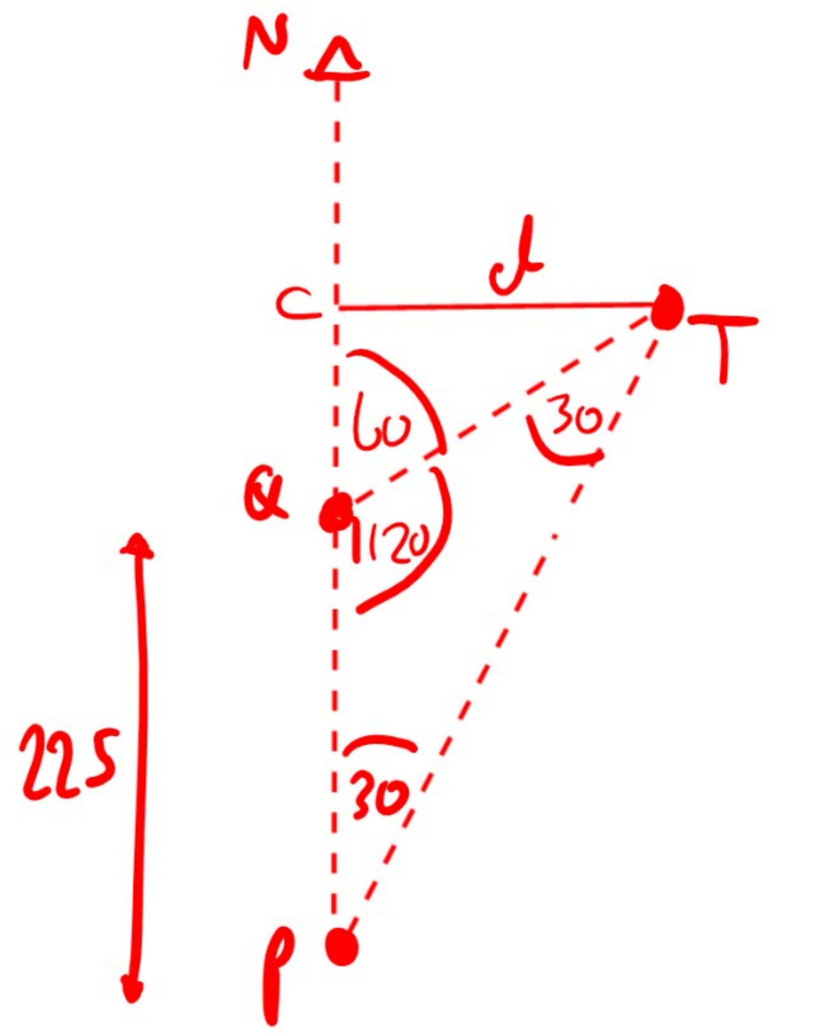
- Unrealistic as the thickness/width of the strips hasn't been taken into account.

Answer: 554.3 mm

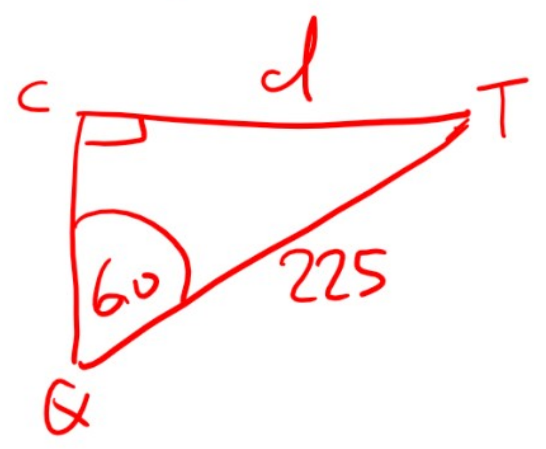
(5 marks)

Q8. A man walks due north along a straight road. When he reaches a point P on this road, he can see a tower T on a bearing of 30° from himself. He continues a further 225 m from point P to the point Q, at which point the tower now lies on a bearing of 60° from his position.

(i) Find the shortest distance of T from the road to 1 decimal place.



$\Rightarrow$ PQT is isoceles, so QT = 120 also



So $\sin(60) = \frac{d}{225}$

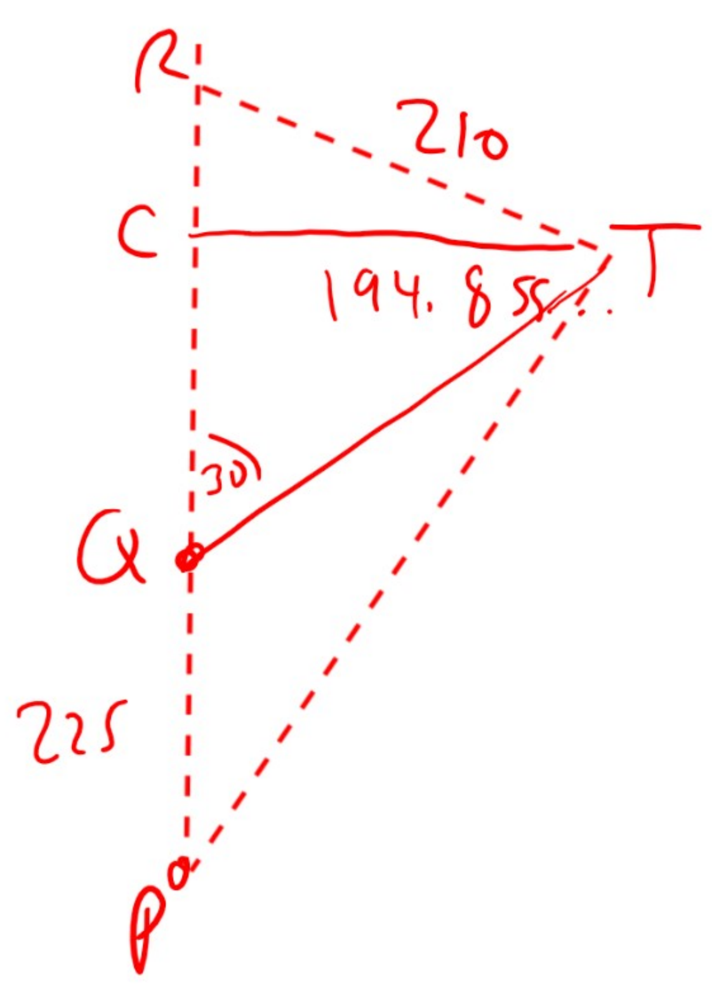
$\Rightarrow d = 194.855...$

Answer: 194.9 m

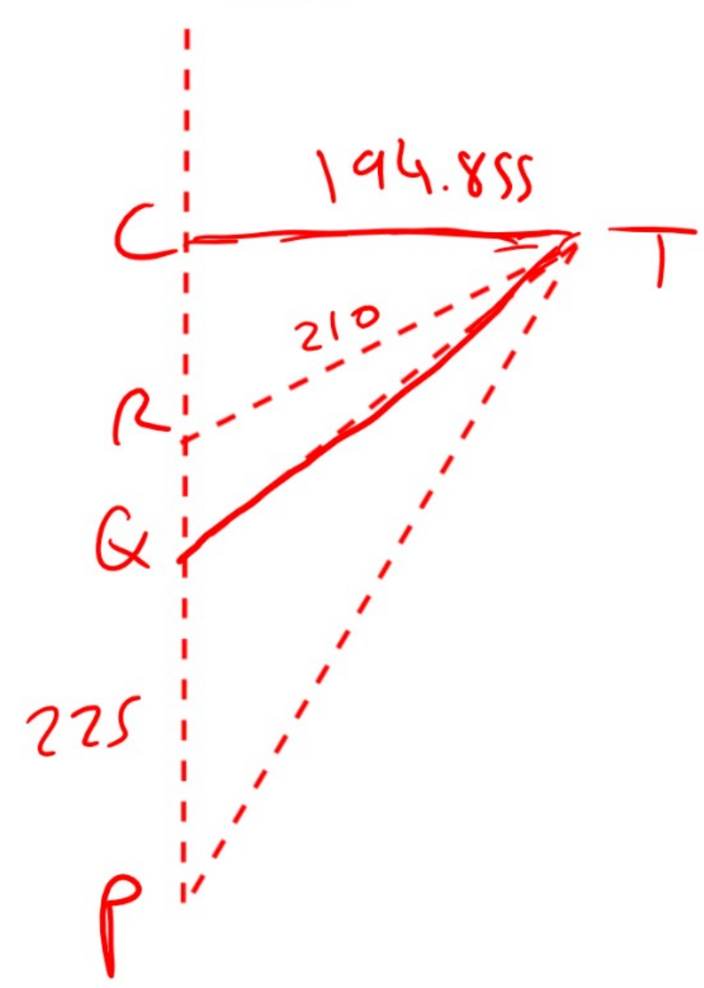
(3 marks)

(ii) The man walks further north to a point R, so that the distance RT is 210 m. Determine the two possible values for the distance PR, correct to the nearest metre.

Case 1



Case 2



- $QC = 337.498..$
- $RC = 78.30$

$\Rightarrow RQ = 259.198$

- $PR = 225 + 259.198$

$= 484.198$

- $QC = \frac{194.855..}{\tan(30)} \Rightarrow QC = 337.498..$
- $RC^2 = 210^2 - (194.855)^2 \Rightarrow RC = 78.30$
- $PR = 225 + 337.498 + 78.30 \Rightarrow PR = 640.80$

Answer: 641m or 484m

(3 marks)

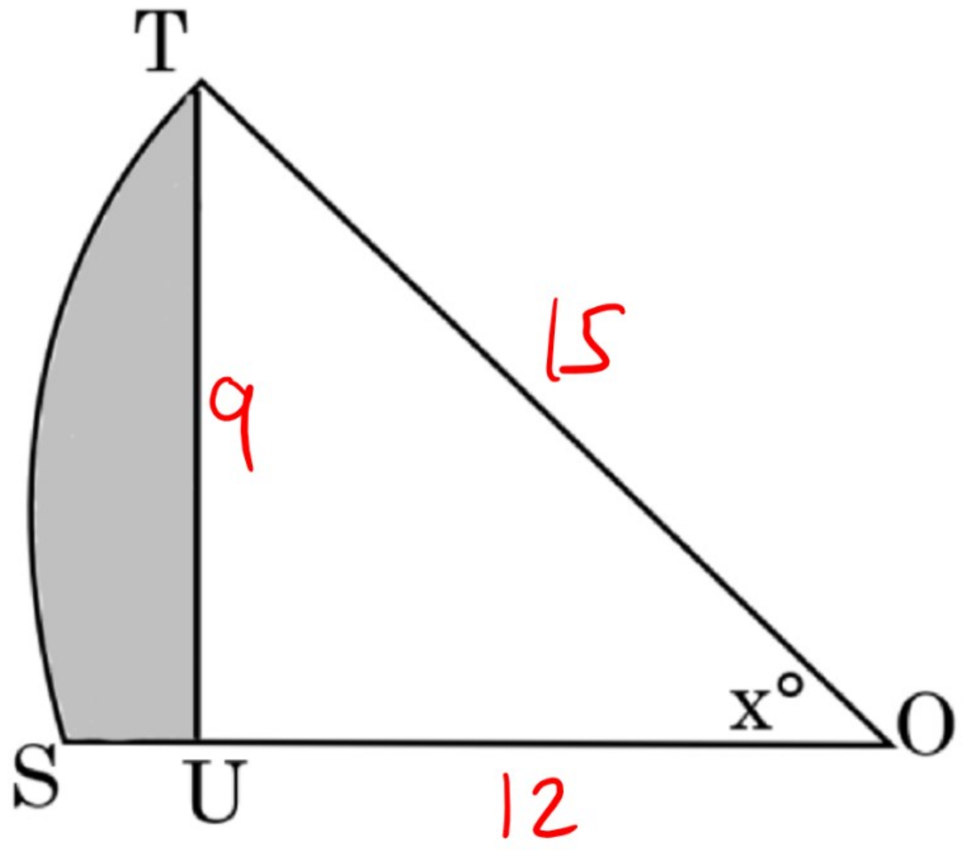
Q9. Below, OST is a sector of a circle. In the triangle:

OU = $2x + 2$,

TU = $2x - 1$ and

OT = $3x$

Find the shaded area correct to 1 decimal place.



• By Pythagoras:

$$(2x+2)^2 + (2x-1)^2 = (3x)^2$$

$$4x^2 + 8x + 4 + 4x^2 - 4x + 1 = 9x^2$$

$$8x^2 + 4x + 5 = 9x^2$$

$$0 = x^2 - 4x - 5$$

$$0 = (x-5)(x+1)$$

$$\Rightarrow \underline{x = 5}, \ -1 \ \text{(reject)}$$

• Area OTU $= \frac{1}{2}(12 \times 9)$

$= \underline{54}$

• $\cos(x) = \frac{12}{15}$

$\Rightarrow x = \cos^{-1}\left(\frac{12}{15}\right)$

$x = 36.869\ldots$

• Area Sector OST $= \frac{36.869}{360} \times \pi \times 15^2$ $\left(\frac{\theta}{360} \times \pi r^2\right)$

$= 72.393\ldots$

• Shaded area = sector − triangle

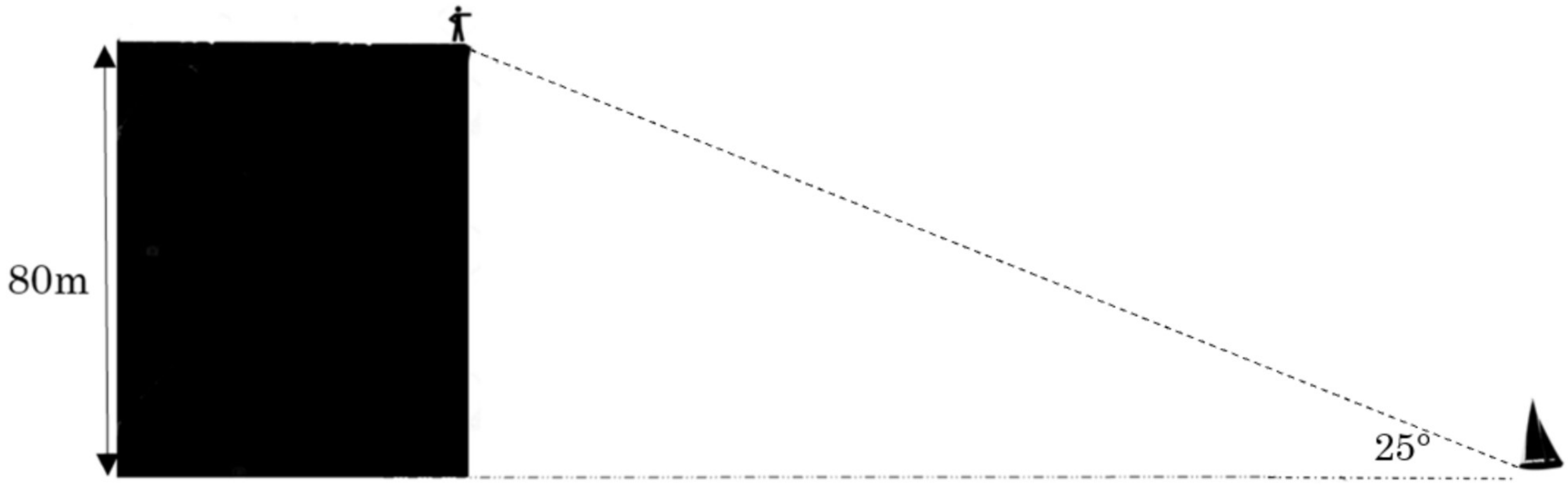
$= 72.393\ldots - 54$

$= 18.393\ldots$

Answer: 18.4 units^2

(6 marks)

Q10. A boat is heading directly for the foot of a vertical cliff at 2.2 m/s. At 11.59am, the position of the boat from the cliff is shown in the diagram:



Rob is standing on top of a cliff, and he will be seen by his friend Tom in the boat once the angle of depression from Rob to Tom is 75°. Estimate what time, to the nearest second, will Rob be seen by Tom.

- let current distance of boat from foot of cliff be d.

$\tan(40) = \frac{80}{d} \Rightarrow d = \frac{80}{\tan(25)}$

$= 171.560\text{m}$

- let distance of boat from cliff when Tom sees Rob be x

$\tan(70) = \frac{80}{x} \Rightarrow x = \frac{80}{\tan(75)} \Rightarrow x = 21.435$

- $\text{time} = \frac{\text{distance}}{\text{speed}} \Rightarrow \text{time} = \frac{171.560 - 21.435}{2.2}$

$= 68.238$ seconds

Answer: 12pm, 8 seconds

(4 marks)

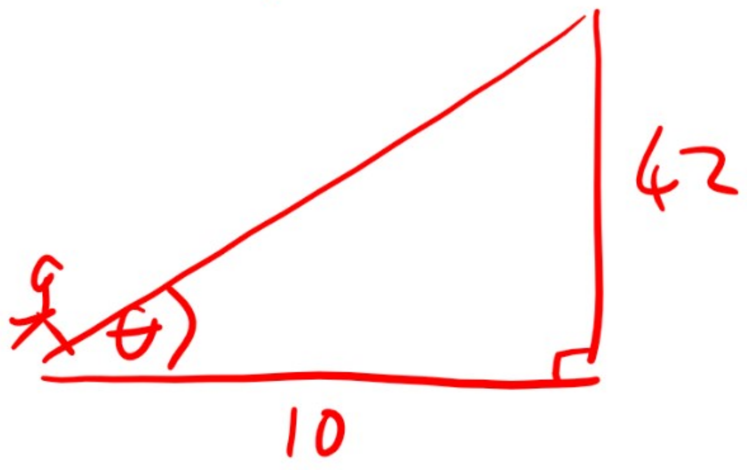
- The height of the man on the boat is not taken into account.
- The wind / currents have not been taken into account.

Answer: ____________

(2 marks)

Q11. Phil is in the grounds of a local church. A friend tells him that the angle of elevation, θ, from his current position to the top of the church tower is such that tan(θ) is $\frac{21}{5}$.
Phil says, “that means the height of the church tower is 21 m”.
Do you agree? Explain your choice.

Not necessarily true – tan(θ) is a ratio, and so the situation could also be like this, for example:



(Here the height of the tower is 42 m)

In other words, $\frac{\text{the height of the tower}}{\text{distance from foot of tower}}$ could be any multiple of $\frac{21}{5}$.

Answer:____________________

(2 marks)