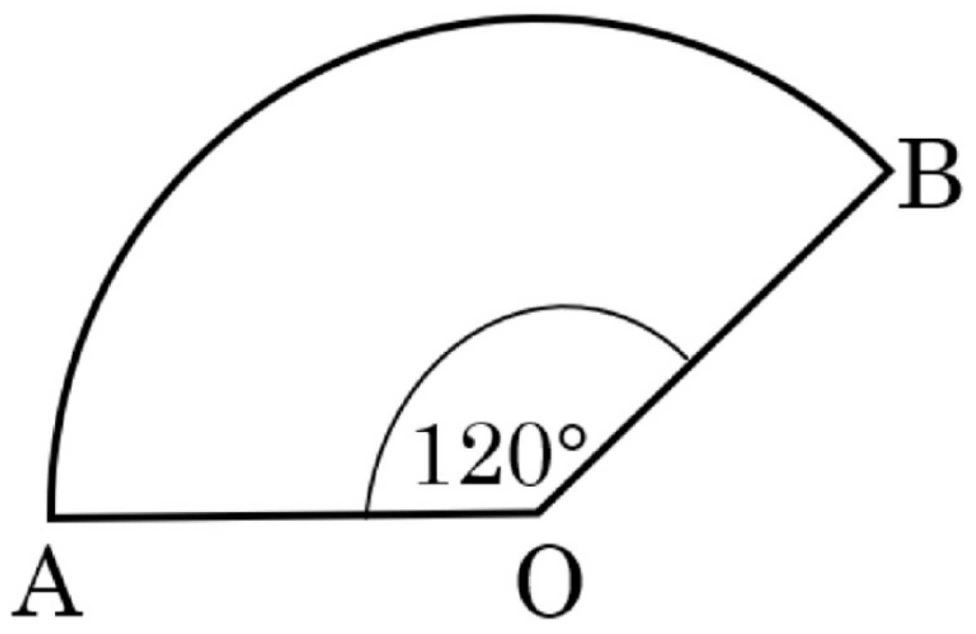
Sectors and Arcs Exam Practice

Q1. In the sector AOB below, OB = 7 cm



a) Find the area of OAB to 1 d.p.

Area $= \frac{120}{360} \times \pi \times 7^2$

$= 51.31...$

Answer: 51.3 cm²

(2 marks)

b) Find the arc length AB to 1 d.p

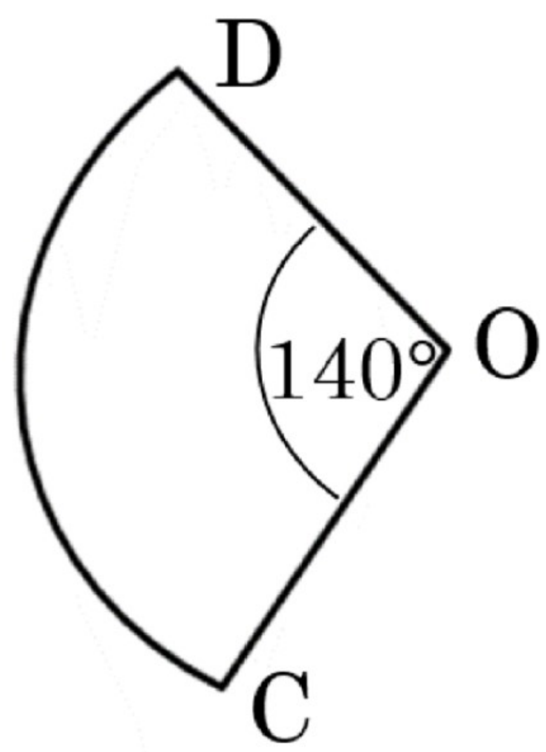
Arc length $= \frac{120}{360} \times 2 \times \pi \times 7$

$= 11.66..$

Answer: 11.7 cm

(2 marks)

Q2. Below OCD is a sector of a circle, with radius 18 cm. Find the perimeter of the shape correct to 2 decimal places.



Perimeter = arc length + 2 × radius

$= \frac{140}{360} \times 2 \times \pi \times 18 + 2 \times 18$

$= 79.982...$

Answer: 79.98 cm

(2 marks)

Q3. OAB is a sector of a circle, which has centre O. The area of the circle is 400 cm². Given that angle AOB is 40°, find the following, leaving your answer in terms of π.

a) the radius of the sector OAB

$$(\text{Area})\ 400 = \frac{40}{360} \times \pi \times r^2$$

$$1144000 = 40\pi r^2$$

$$3600 = \pi r^2$$

$$\frac{3600}{\pi} = r^2$$

$$r = \sqrt{\frac{3600}{\pi}}$$

$$r = 33.85..$$

Answer: 33.9cm

(3 marks)

b) the perimeter of the sector OAB

perimeter = arc length + 2 × radius

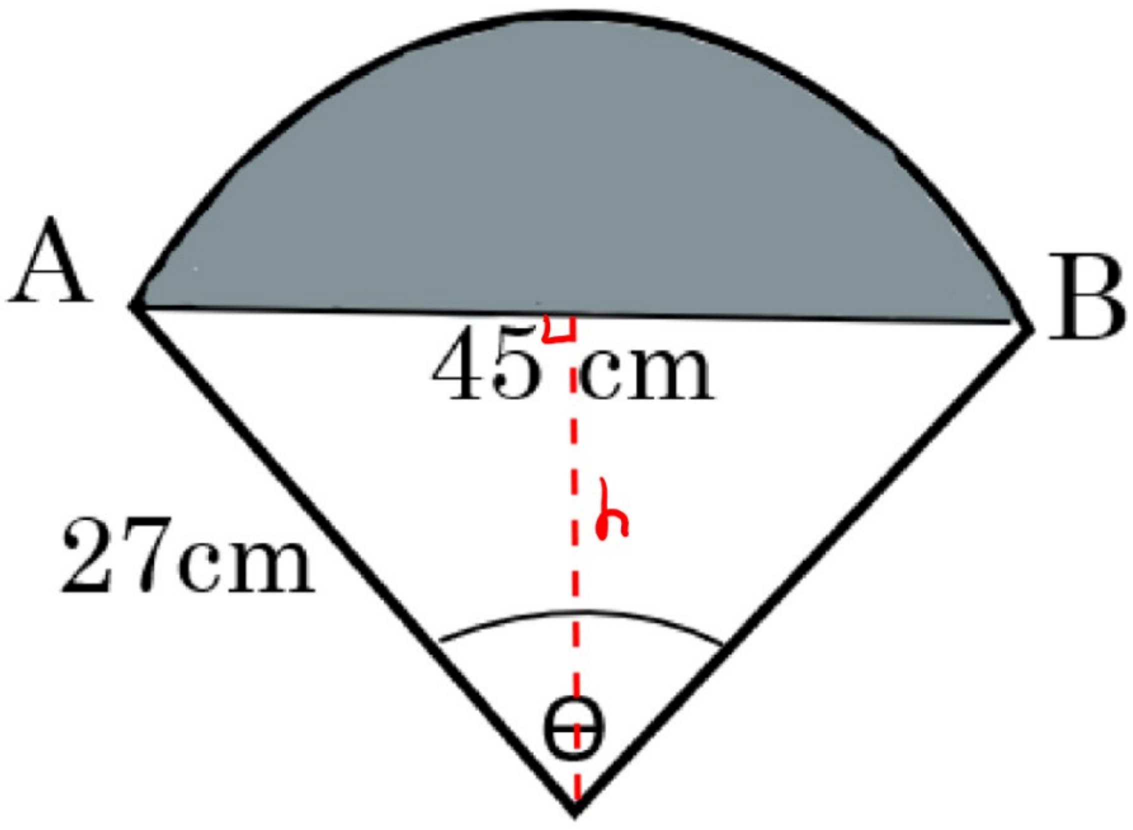
$$= \frac{40}{360} \times 2 \times \pi \times 33.85\ldots + 2(33.85\ldots)$$

$$= 91.33\ldots$$

Answer: 91.3 cm

(3 marks)

Q4. AOB is a sector, AB is a chord, and angle AOB = θ.



Using cosine rule to find θ,

$a^2 = b^2 + c^2 - 2bc\cos\theta$

$45^2 = 27^2 + 27^2 - 2(27)(27)\cos\theta$

$2025 = 729 + 729 - 1458\cos\theta$

$567 = -1458\cos\theta$

$-0.388\ldots = \cos\theta$

$\theta = 67.1146\ldots$

a) Find the length of the arc AB

$$\text{arc length} = \frac{67.1146\ldots}{360} \times 2\pi \times 27$$

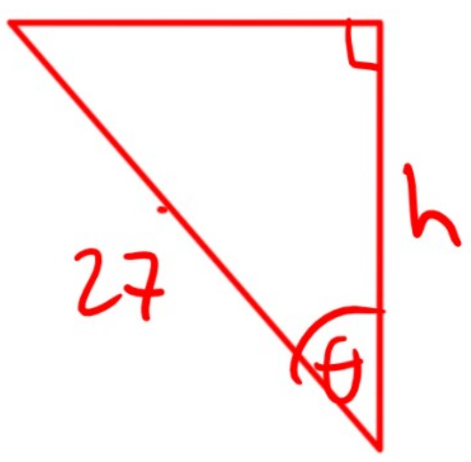
$$= 31.627\ldots$$

Answer: 31.6 cm

(5 marks)

b) Find the shortest distance from O to the chord.

we find side h shown above.



where $\theta = \frac{31.627}{2}$

$= 15.813\ldots$

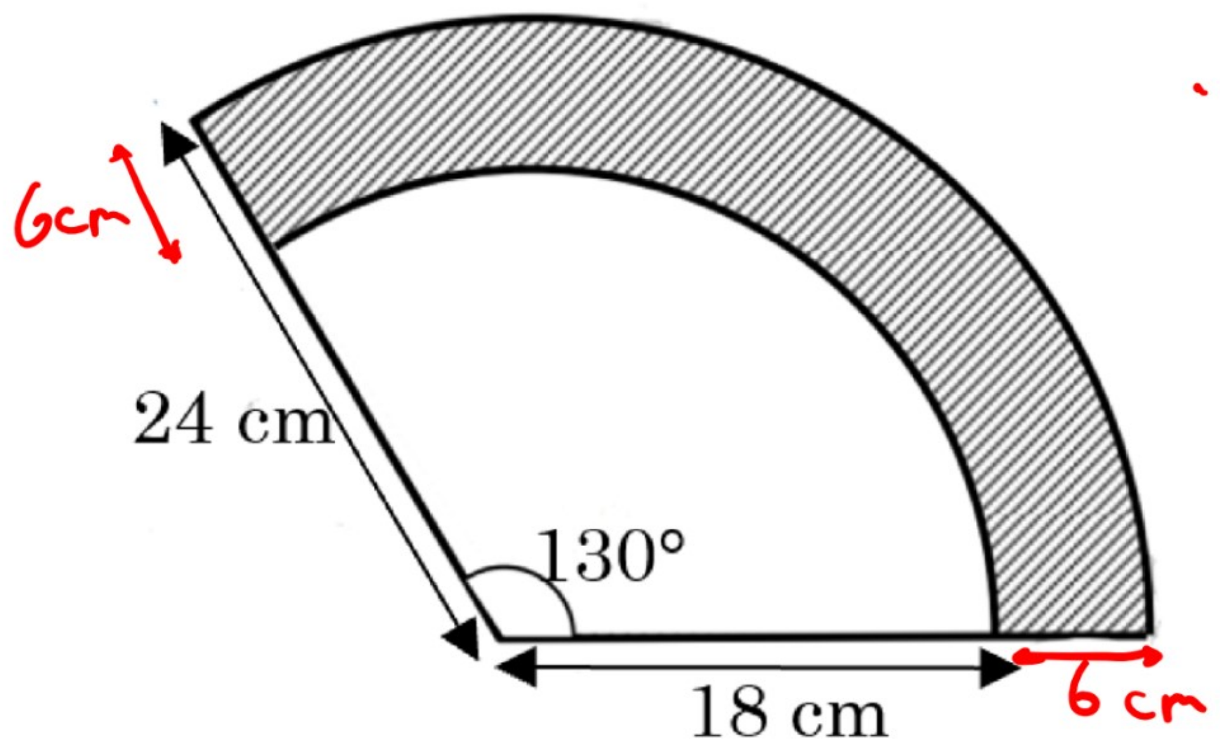
$$\cos(15.813\ldots) = \frac{h}{27}$$

$h = 27\cos(15.813\ldots) \Rightarrow h = 25.979\ldots$

Answer: 25.98 cm (2 d.p.)

(3 marks)

Q5. The picture shows part of two concentric circles, of radii 18 and 24 cm. Find the area and perimeter of the shaded region.



• Shaded area = large sector − small sector

$= \frac{130}{360} \times \pi \times 24^2 - \frac{130}{360} \times \pi \times 18^2$

$= 285.88\ldots$

$= 285.9 \text{ cm}^2$

• perimeter of shaded area = large sector arc + small sector arc + 2×6

$= \frac{130}{360} \times 2\pi \times 24 + \frac{130}{360} \times 2\pi \times 18 + 12$

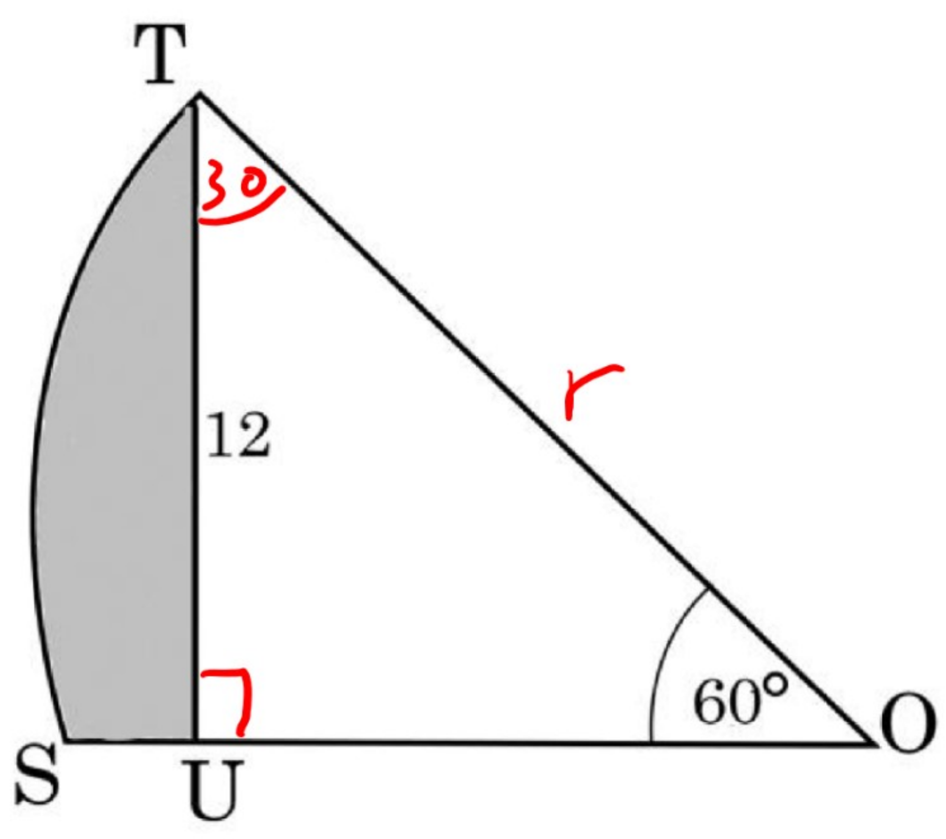
$= 107.294\ldots$

$= 107.3 \text{ cm}$

Answer: 107.3cm ; 285.9cm²

(3 marks)

Q6. Below, OST is sector, and angle OUT is 90°. Find the area of the shaded region to 1 decimal place.



$\sin(60) = \frac{12}{r}$

$r = \frac{12}{\sin(60)}$

$= \frac{24}{\sqrt{3}}$

• area shaded region = area sector − area △OUT

• using $\frac{1}{2}ab\sin C$ for area of △OUT,

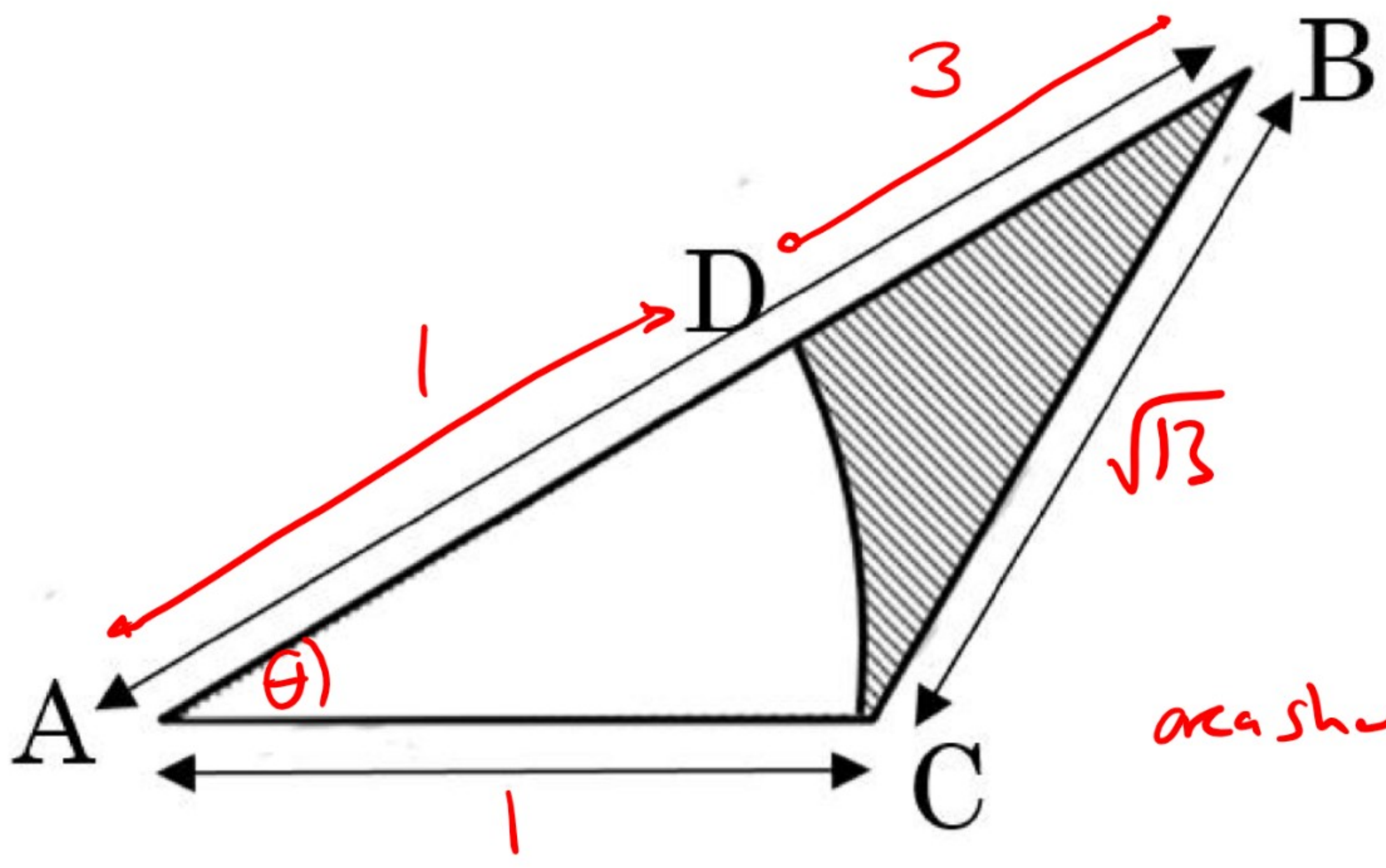
$= \frac{60}{360} \times \pi \times \left(\frac{24}{\sqrt{3}}\right)^2 - \frac{1}{2} \times 12 \times \left(\frac{24}{\sqrt{3}}\right) \sin(30)$

$= 58.9617\ldots$

Answer: 58.96 units²

(3 marks)

Q7. In triangle ABC below, AB = 4, AC = 1 and BC = $\sqrt{13}$. ACD is a sector with radius 1.



• First find θ using cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

$(\sqrt{13})^2 = 4^2 + 1^2 - 2(4)(1)\cos(\theta)$

$13 = 17 - 8\cos\theta$

$\cos\theta = \frac{-4}{-8} \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60°$

area shaded region = area ΔABC − area ADC

$= \frac{1}{2}(4)(1)\sin 60 - \frac{60}{360}\times\pi(1^2)$

$= 1 - \frac{1}{6}\pi$

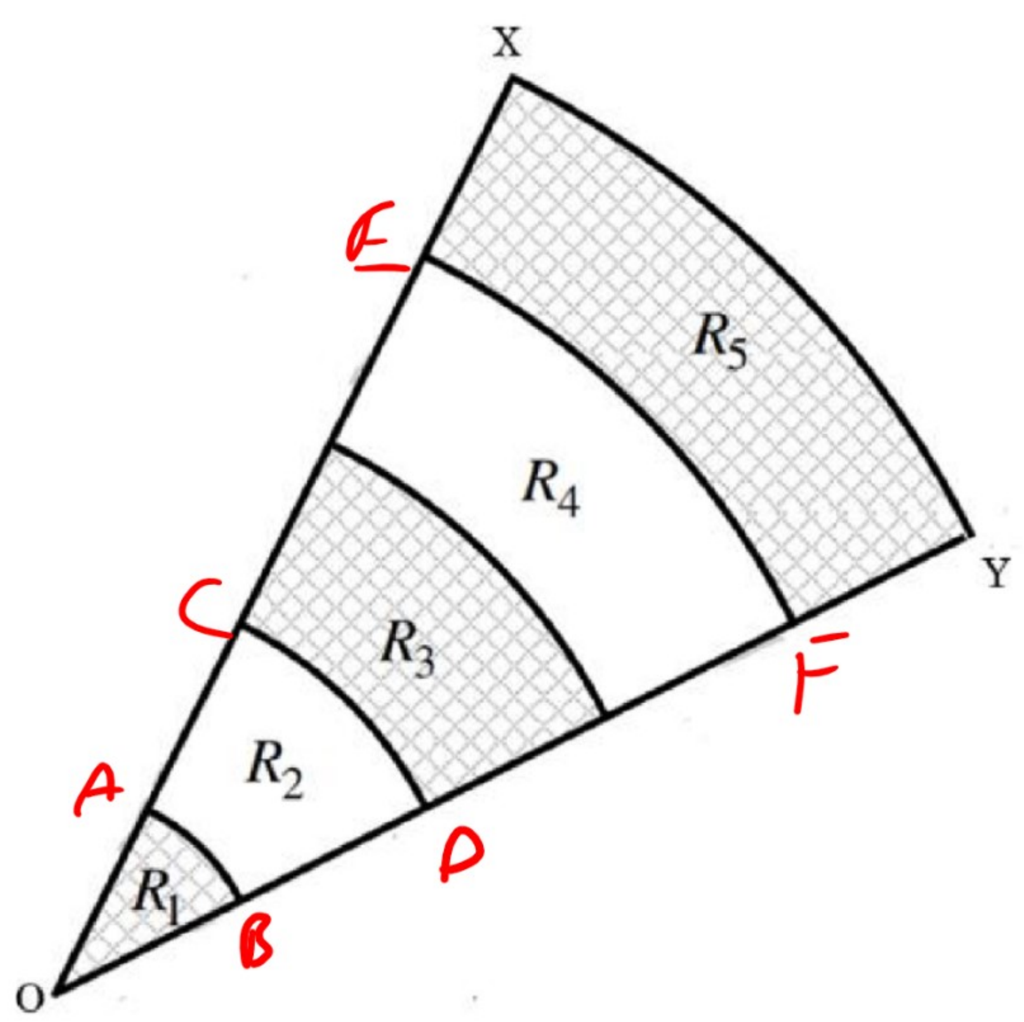
$= \frac{6-\pi}{6}$

Find an exact expression for the shaded area.

Answer: $\frac{6-\pi}{6}$

(6 marks)

Q8. OXY is a sector of a circle, containing equally spaced sectors within in it. Find the ratio of the area of region R_2 to the area of region R_5.



• let OA = r, $X\hat{O}Y = \theta$

• area R_2 = Area OAB − Area R_1

$= \frac{\theta}{360}\times\pi\times(2r)^2 - \frac{\theta}{360}\times\pi\times r^2$

$= \frac{\theta}{360}\left[4\pi r^2 - \pi r^2\right]$

$= \frac{\theta}{360}(3\pi r^2)$

• area R5 = area OXY − area OEF

$= \frac{\theta}{360}\times\pi(5r)^2 - \frac{\theta}{360}\times\pi\times(4r)^2$

$= \frac{\theta}{360}\left[25\pi r^2 - 16\pi r^2\right]$

$= \frac{\theta}{360}(9\pi r^2)$

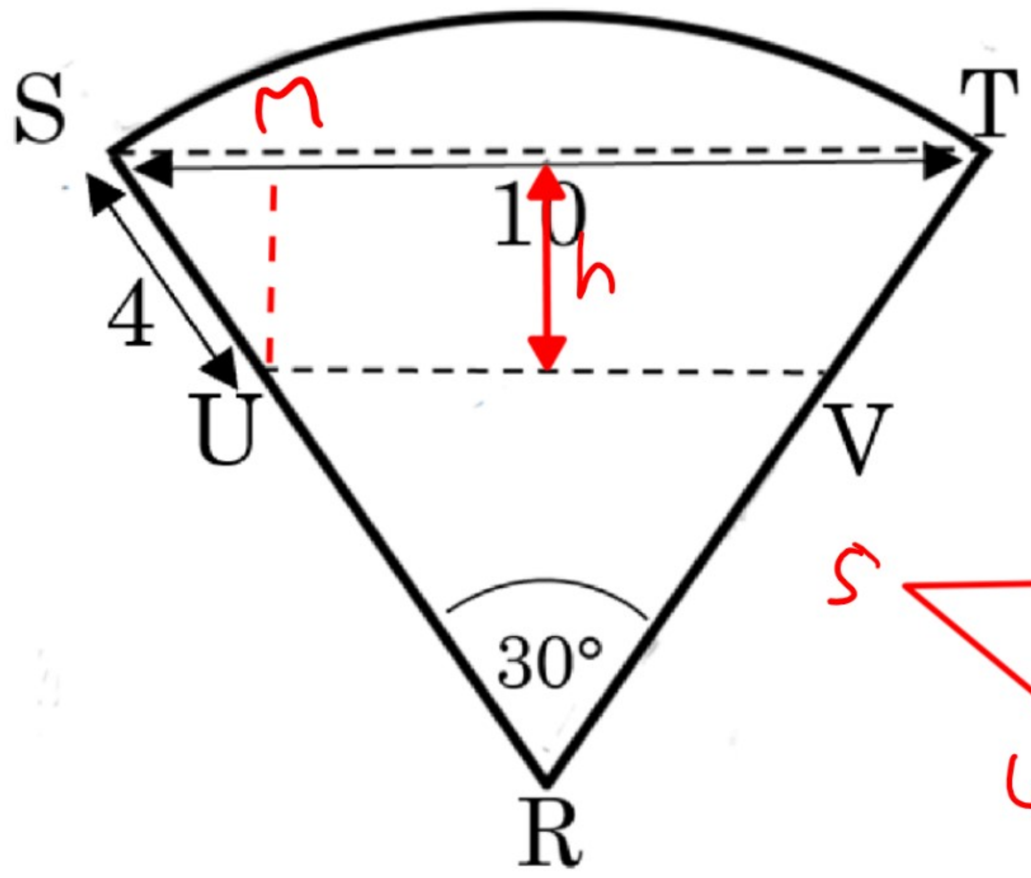
$\therefore$ area R_5 : area R_2

= 3 : 1

Answer: 3 : 1

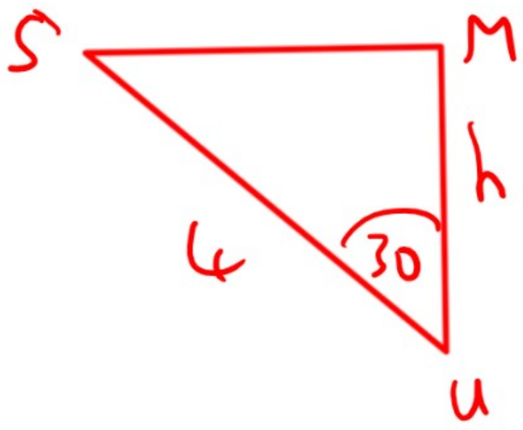
(6 marks)

Q9. In the sector RST, ST is a chord, where ST = 10, and SU = 4.



STUV is a trapezium
$\left(\text{Area} = \frac{1}{2}(a+b)h\right)$

$\therefore$ Area $STUV = \frac{1}{2}(ST + UV)h$



$\cos(30) = \frac{h}{4} \Rightarrow h = 4\cos(30)$

Find the area of the region enclosed by the dotted lines.

$(SM)^2 = (SU)^2 - (MU)^2$ (Pythagoras' Theorem)

$\Rightarrow (SM)^2 = 4^2 - (4\cos(30))^2$

$\Rightarrow \quad = 4$

$\Rightarrow SM = 2$

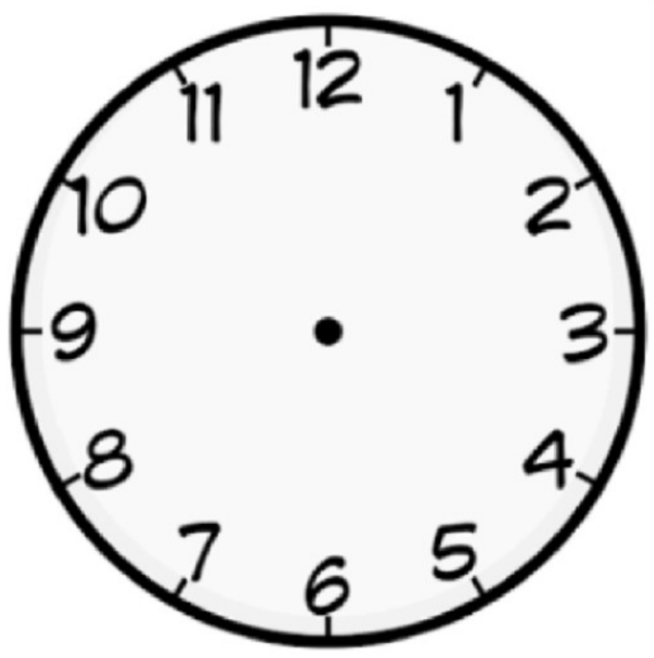
- By symmetry of the sector, $UV = ST - 2 - 2$
$= 6$

$\therefore$ area $= \frac{1}{2}(10+6)4\cos(30)$
$= 27.71\ldots$

Answer: 27.71 cm^2

(5 marks)

Q10. Hands are attached to the clock-face below, so the time shown is 10.00 am. When the time reaches 5.00pm, the tip of the hour hand has travelled 34 cm.



- Each hour is $\frac{360°}{12} = 30°$
- Total angle travelled by long hand is $7 \times 30° = 210°$
- let r = length of hour hand

Work out the length of the hour hand to 1 d.p.

$34 = \frac{210°}{360°} \times 2\pi \times r$ (using arc length formula)

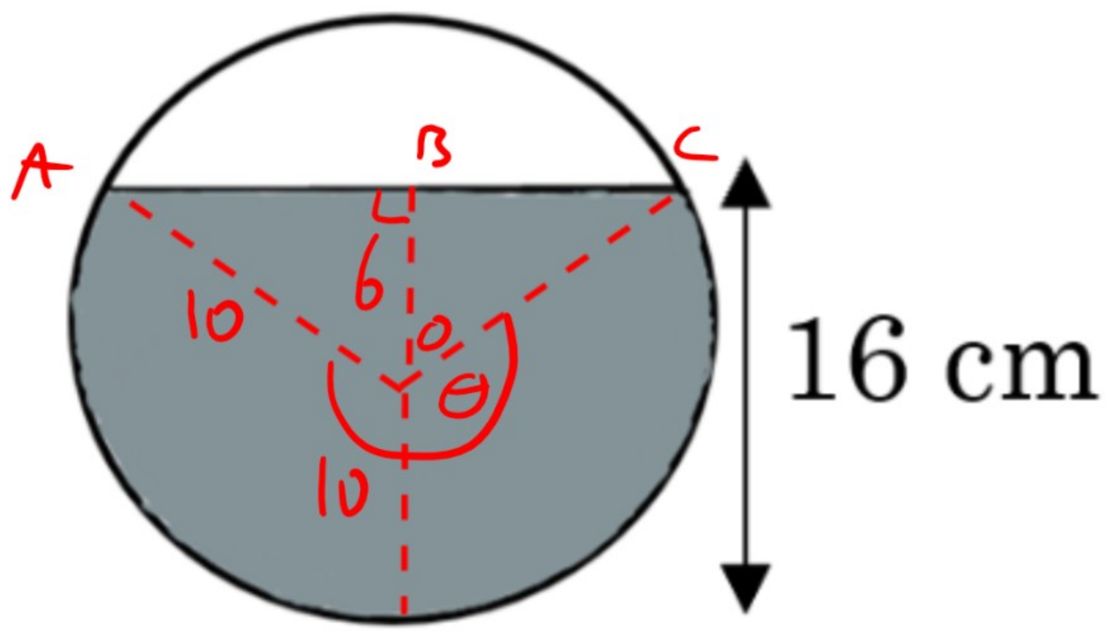
$12240 = 210 \times 2\pi r$

$\frac{12240}{210 \times 2\pi} = r \Rightarrow r = 9.27\ldots$

Answer: 9.3 cm

(4 marks)

Q11. A circular oil pipe, with diameter 20 cm, has cross-section below.
If oil flows at a constant height through the pipe at 0.25 m/s,
find the volume of oil which passes through the pipe in 1 hour, to 3 s.f.



$r = 10$

$1000 \text{ cm}^3 = 1$ litre

- Area of shaded region = area $\triangle OAC$ + major sector AOC

- Area $\triangle OAC = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times AC \times 6$,
where $AC = 2 \times AB$
$= 2 \times 8$
$= 16$

(By Pythagoras: $(AB)^2 + 6^2 = 10^2$
$\Rightarrow (AB)^2 + 36 = 100$
$\Rightarrow (AB)^2 = 64$
$AB = \sqrt{64}$
$= 8$)

$\Rightarrow$ Area $\triangle OAC = \frac{1}{2} \times 16 \times 6$
$= 48 \text{ cm}^2$

- Angle $\theta = 360 - 2 \times A\hat{O}B$
where $A\hat{O}B = \cos^{-1}\left(\frac{6}{10}\right)$
$= 53.130...$
$\Rightarrow \theta = 360 - 106.260...$
$= 253.739...$

- Area major sector $AOC = \frac{253.739}{360} \times \pi \times 10^2$
$= 221.429...$

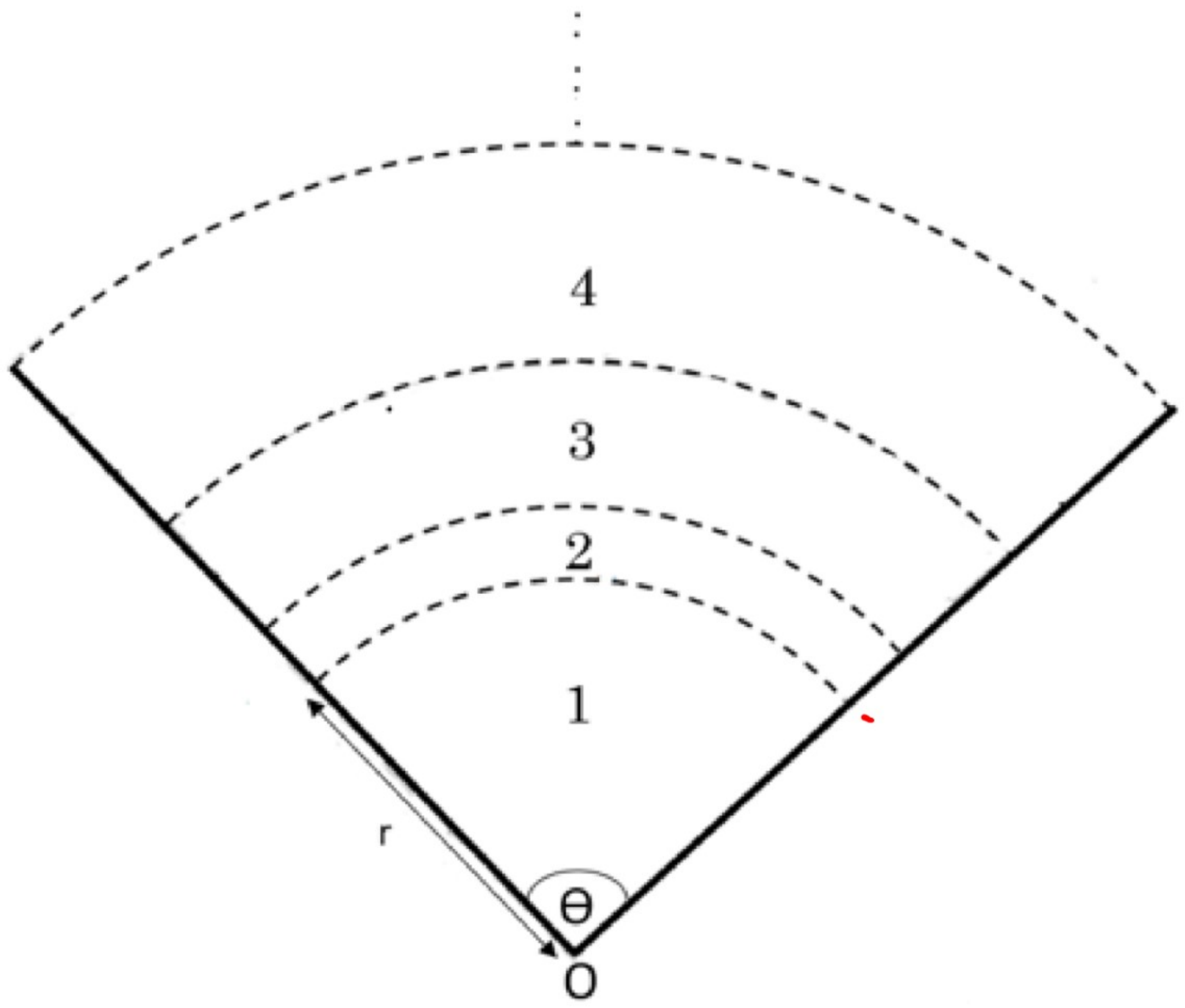
$\therefore$ area shaded cross section $= 221.429.... + 48$
$= 269.429.. \text{ cm}^2$

- volume per second $= 269.429 \times 25 \text{ cm}^3$
- volume per hour $= (269.429 \times 25) \times 3600$
$= 24248676.92 \text{ cm}^3$

Answer: 24,249 litres

(7 marks)

Q12. For a sport, an area is formed from concentric circles all having centre O. Region 1 ends r metres from O, with each adjacent area finishing 15% further from O than the previous area.



Find an expression for the area of region N in terms of r, ϴ, simplifying your answer as far as possible.

- Area Region N = Area of sector containing regions 1 to N − Area of sector containing regions 1 to (N−1)
- radius of sector containing 1 to N is $r \times 1.15^{N-1}$

$\Rightarrow$ its area is $\frac{\theta}{360} \times \pi (r \times 1.15^{N-1})^2$

- radius of sector containing 1 to N−1 is $r \times 1.15^{N-2}$

$\Rightarrow$ its area is $\frac{\theta}{360} \times \pi (r \times 1.15^{N-2})^2$

- Required area $= \frac{\theta}{360}\pi\left[r^2 \times 1.15^{N+1} - r^2 \times 1.15^{N}\right]$

$= \frac{\theta}{360}\pi r^2 1.15^N[1.15 - 1]$

Answer: $\frac{\theta}{2400}\pi r^2 1.15^N$

(6 marks)