



Quadratic Simultaneous Equations Exam Practice

Q1. Solve the simultaneous equations:

$$x^2 + y^2 = 17 \quad (1)$$

$$2x + y = 2 \quad (2) \Rightarrow y = 2 - 2x \quad \text{in } (1):$$

$$x^2 + (2 - 2x)^2 = 17$$

$$x^2 + 4 - 8x + 4x^2 = 17$$

$$5x^2 - 8x - 13 = 0$$

$$(5x - 13)(x + 1) = 0$$

$$x = \frac{13}{5}, \quad x = -1 \quad \text{in } (2)$$

$$y = -3\frac{1}{5}, \quad y = 4$$

$$x = \frac{13}{5}, \quad y = -3\frac{1}{5}$$

$$\text{Answer: } \underline{x = -1, y = 4}$$

(5 marks)

Q2. Solve the simultaneous equations:

$$x^2 + 4y^2 = 298$$

$$4y + 3x = 43 \quad \Rightarrow \quad x = \frac{43 - 4y}{3}$$

[You may find the factorisation $833 = 49 \times 17$ useful.]

$$\left(\frac{43 - 4y}{3}\right)^2 + 4y^2 = 298$$

$$\frac{1849 - 344y + 16y^2}{9} + 4y^2 = 298$$

$$1849 - 344y + 16y^2 + 36y^2 = 2682$$

$$52y^2 - 344y - 833 = 0$$

$$(26y + 49)(2y - 17) = 0$$

$$y = -\frac{49}{26}, \quad y = 8.5 \quad \text{in } (2)$$

$$x = \frac{219}{13}, \quad x = 3$$

$$x = \frac{219}{13}, \quad y = -\frac{49}{26}$$

$$\text{Answer: } \underline{x = 3, y = 8.5}$$

(5 marks)



Q3. Solve the simultaneous equations, correct to 2 decimal places:

$$2x^2 + y^2 = 59 \quad (1)$$

$$3x + 7y = 36 \quad (2) \Rightarrow y = \frac{36 - 3x}{7} \quad \text{in } (1)$$

$$2x^2 + \left(\frac{36 - 3x}{7}\right)^2 = 59$$

$$2x^2 + \frac{1296 - 216x + 9x^2}{49} = 59$$

$$98x^2 + 1296 - 216x + 9x^2 = 2891$$

$$107x^2 - 216x - 1595 = 0$$

$$x = 5.0, \quad x = -2.981\dots \quad \text{in } (2)$$

$$y = 3.0, \quad y = 6.420\dots$$

$$x = 5.00, \quad y = 3.00$$

$$x = -2.98, \quad y = 6.42$$

Answer:

(5 marks)

Q4. Find all the solutions to the pair of simultaneous equations:

$$y^2 - 4x = 13 \quad (1)$$

$$y - 2x = -1 \quad (2) \Rightarrow y = 2x - 1 \quad \text{in } (1)$$

$$(2x - 1)^2 - 4x = 13$$

$$4x^2 - 4x + 1 - 4x = 13$$

$$4x^2 - 8x - 12 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3)$$

$$x = -1, \quad x = 3 \quad \text{in } (2)$$

$$y = -3, \quad y = 5$$

$$x = 1, \quad y = -3$$

$$x = 3, \quad y = 5$$

Answer:

(6 marks)



Q5. Find all the solutions to the pair of simultaneous equations:

$$x^2 - 2x = y + 7 \quad (1)$$

$$y - x = 3 \quad (2) \Rightarrow y = x + 3 \quad \text{in } (1)$$

$$x^2 - 2x = x + 3 + 7$$

$$x^2 - 3x - 10 = 0$$

$$(x + 2)(x - 5) = 0$$

$$x = -2, x = 5 \quad \text{in } (2)$$

$$y = 1, y = 8$$

Answer: $x = -2, y = 1$
 $x = 5, y = 8$
(6 marks)

Q6. Solve the simultaneous equations:

$$3s + t = 8 \quad (1) \Rightarrow t = 8 - 3s \quad \text{in } (2)$$

$$3s^2 + t^2 = 28 \quad (2)$$

$$3s^2 + (8 - 3s)^2 = 28$$

$$3s^2 + 64 - 48s + 9s^2 = 28$$

$$12s^2 - 48s + 36 = 0$$

$$s^2 - 4s + 3 = 0$$

$$(s - 3)(s - 1) = 0$$

$$s = 3, s = 1 \quad \text{in } (1)$$

$$t = -1 \quad t = 5$$

Answer: $s = 3, t = -1$
 $s = 1, t = 5$
(5 marks)



Q7. Solve the simultaneous equations:

$$s + 2t = 3 \quad (1) \Rightarrow s = 3 - 2t \quad \text{in } (2)$$

$$3st + s^2 = 10 \quad (2)$$

$$3(3-2t)t + (3-2t)^2 = 10$$

$$9t - 6t^2 + 9 - 12t + 4t^2 = 10$$

$$-2t^2 - 3t - 1 = 0$$

$$2t^2 + 3t + 1 = 0$$

$$(2t + 1)(t + 1)$$

$$t = -\frac{1}{2}, t = -1 \quad \text{in } (1)$$

$$s = 4, s = 5$$

Answer: $t = -\frac{1}{2}, s = 4$
 $t = -1, s = 5$

(6 marks)



Applied Mixed Practice Problems

Q8. Given the pair of simultaneous questions

$$a + 4b = 2$$

$$3b^2 + ab = -11$$

show that $b^2 - 2b - 11 = 0$

a) show that $b^2 - 2b - 11 = 0$

$$3b^2 + (2-4b)b = -11$$

$$3b^2 + 2b - 4b^2 = -11$$

$$-b^2 + 2b = -11$$

$$0 = b^2 - 2b - 11 \quad (\text{done})$$

Answer: _____
(2 marks)

b) Hence solve the pair of simultaneous equations, giving your answers in exact form.

$$b^2 - 2b - 11 = 0$$

$$b = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2}$$

$$= \frac{2 \pm \sqrt{48}}{2}$$

$$= 1 \pm 2\sqrt{3} \quad \text{in (1)}$$

$$\Rightarrow \begin{aligned} a &= 8\sqrt{3} - 2, & b &= 1 - 2\sqrt{3} \\ a &= -2 - 8\sqrt{3}, & b &= 1 + 2\sqrt{3} \end{aligned}$$

Answer: $a = 8\sqrt{3} - 2, b = 1 - 2\sqrt{3}$
 $a = -2 - 8\sqrt{3}, b = 1 + 2\sqrt{3}$

(3 marks)



Q9. Sandy throws a ball in a sport-hall.

- The height h of the ball in metres can be modelled by the quadratic equation, $h = -0.05x(x - 32)$, where x is the horizontal distance travelled.
- The roof of the sports-hall can be modelled by the linear equation, $y - 0.15x - 8 = 0$.

a) Show that the model predicts that the ball hits the roof of the sports-hall.

$$\cdot y = 0.15x + 8$$

$$\cdot h = -0.05x^2 + 2x$$

If ball hits roof, $y = h$ for some x

$$\Rightarrow -0.05x^2 + 2x = 0.15x + 8$$

$$\Rightarrow 0.05x^2 - 1.85x + 8 = 0 \Rightarrow 5x^2 - 185x + 800 = 0$$

$$\Rightarrow x^2 - 37x + 160 = 0$$

$$\Rightarrow (x - 5)(x - 32) = 0$$

The ball hits the roof when it has travelled 5m horizontally.

Answer: _____
(4 marks)

b) State the horizontal distance from Sandy to the point directly below the roof of the point of contact. Give your answer to 1 decimal place.

(The solution $x = 32$ does not apply, as the ball will have bounced off the roof at $x = 5$)

Answer: 5m
(2 marks)



①

Q10. A curve has equation, $y^2 + 2x + 6 = 0$ and a line has equation $y + x + 7 = 0$. ②

a) Find the co-ordinates of any points where the line and the curve meet.

② $\Rightarrow x = -y - 7$, in ① :

$$y^2 + 2(-y - 7) + 6 = 0$$

$$\rightarrow y^2 - 2y - 8 = 0$$

$$(y - 4)(y + 2) = 0$$

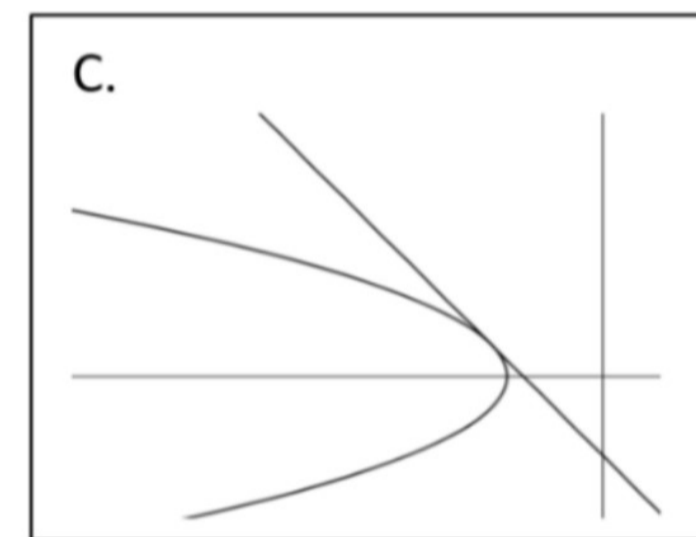
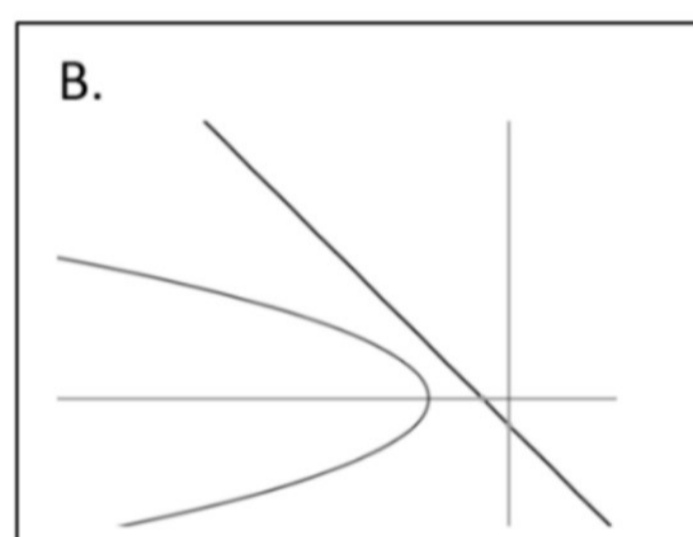
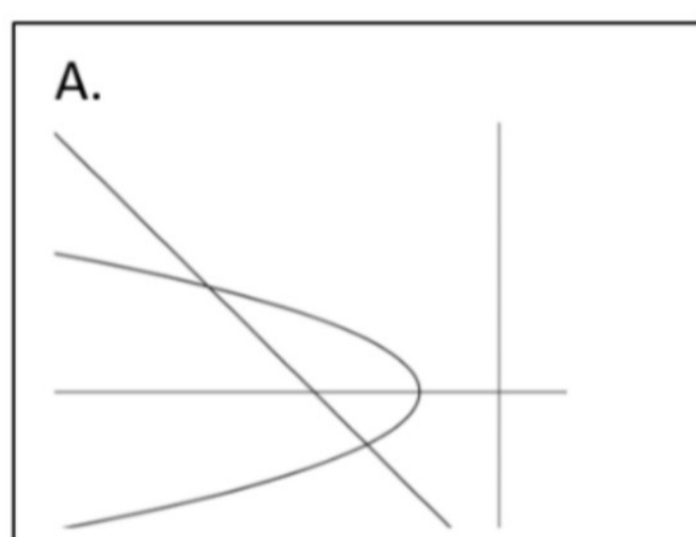
$$y = 4, y = -2 \text{ in ②}$$

$$x = -11, x = -5$$

$$x = -11, y = 4$$
$$x = -5, y = -2$$

Answer: _____
(5 marks)

b) Which of these diagrams below represent the curve and the line?
You must explain your choice.



① — there are 2 distinct points of intersection, corresponding to the 2 distinct solutions in part (a)

Answer: ①
(1 mark)



Q11. Given the pair of simultaneous equations, find the values of x :

$$y + 4x = 2 \quad (1)$$

$$3x^2 + xy = -11. \quad (2)$$

Give your answers in the form $a \pm b\sqrt{3}$ where a, b are whole numbers.

$$y = 2 - 4x \text{ in (2) :}$$

$$3x^2 + x(2 - 4x) = -11$$

$$3x^2 + 2x - 4x^2 = -11$$

$$x^2 - 2x - 11 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2}$$

$$= \frac{2 \pm \sqrt{48}}{2}$$

$$= \frac{2 \pm 4\sqrt{3}}{2}$$

$$= 1 \pm 2\sqrt{3}$$

Answer: $1 \pm 2\sqrt{3}$
(5 marks)