# Quadratic Sequence Exam Practice

Q1. a) Write down the next two terms in the quadratic sequence:

3, 7, 13, 21, 31, 43, 57

4, 6, 8, 10, 12, 14

2, 2, 2, 2, 2

Answer: 43, 57

(2 marks)

b) Find an expression for the $n^{th}$ term of this sequence.

$a = \frac{1}{2}(2)$ ($\frac{1}{2}$ of 2nd difference)

$\Rightarrow a = 1$

| $n$ | 1 | 2 | 3 | .... |
|---|---|---|---|---|
| $an^2$ | 1 | 4 | 9 | .... |
| Target | 3 | 7 | 13 | |
| $3n+c$ | 2 | 3 | 4 | |

$\Rightarrow n^2 + n + 1$

Answer: $n^2+n+1$

(2 marks)

Q2. a) Write down the next two terms in the quadratic sequence:

13, 10, 2, -11, -29, -52, -75

-3, -8, -13, -18, -23, -27

-5, -5, -5, -5, -5

Answer: $-52, -75$

(2 marks)

b) Find an expression for the $n^{th}$ term of this sequence.

$a = -\frac{5}{2}$

| $n$ | 1 | 2 | 3 | .... |
|---|---|---|---|---|
| $an^2$ | $-\frac{5}{2}$ | $-10$ | $-\frac{45}{2}$ | |
| Target | 13 | 10 | 2 | |
| $bn+c$ | $\frac{31}{2}$ | 20 | $\frac{49}{2}$ | |

$\Rightarrow \frac{9}{2}n + \frac{31}{2}$

Answer: $-\frac{5}{2}n^2 + \frac{9}{2}n + \frac{31}{2}$

(2 marks)

Q3. a) Write down the next two terms in the quadratic sequence:

5, 8, 13, 20, 29, 40, 53

3, 5, 7, 9, 11, 13

2, 2, 2, 2, 2

Answer: _____

(2 marks)

b) Find an expression for the $n^{th}$ term of this sequence.

$a = \frac{1}{2}(2)$

$a = 1$

| $n$ | 1 | 2 | 3 ... |
|---|---|---|---|
| $an^2$ | 1 | 4 | 9 |
| Target | 5 | 8 | 13 ... |
| $bn+c$ | 4 | 4 | 4 |

$\Rightarrow b = 0, c = 4$

Answer: $n^2 + 4$

(2 marks)

Q4. a) Write down the 1st, 2nd and 10th terms of the quadratic sequence which has $n^{th}$ term given by:

$$n^2 + 9$$

1st: $1^2 + 9 = 10$

2nd: $2^2 + 9 = 13$

10th: $10^2 + 9 = 109$

Answer: 10, 13, 109

(2 marks)

b) Is the term 729 in the sequence? You must show your reasoning.

$n^2 + 9 = 729$

$n^2 = 720$

$n = \sqrt{720}$ ≠ whole number, so no

Answer: no.

(2 marks)

Q5. a) Write down the 3rd , 5th and 20th terms of the quadratic sequence which has nth term given by:

$$n^2 - 12n + 9$$

3rd : $3^2 - 12\times3 + 9 = -18$

5th : $5^2 - 12\times5 + 9 = -26$

20th : $20^2 - 12\times20 + 9 = 169$

Answer: $-18, -26, 169$

(2 marks)

b) Is the term 336 in the sequence? You must show your reasoning.

$n^2 - 12n + 9 = 336$

$n^2 - 12n - 327 = 0$

quadratic formula: $n = 6 \pm 11\sqrt{3}$, so not whole.

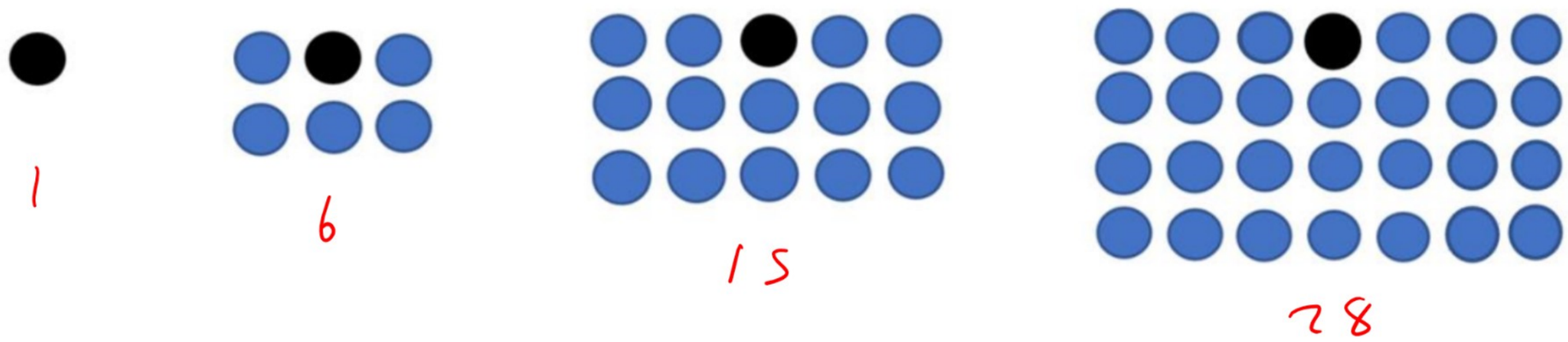
So not in sequence

Answer: no

(2 marks)

## Applied Mixed Practice Problems

Q6. Here is a pattern made from circular dots:




a) Find an expression for the number of dots in the $n^{th}$ pattern.

1 6 15 28

5 9 13

4 4

$a = \frac{1}{2}(4)$

$a = 2$

$5n+c$ is $-n$

| $n$ | 1 | 2 | 3 | ... |
|---|---|---|---|---|
| $an^2$ | 2 | 8 | 18 | ... |
| Target | 1 | 6 | 15 | |
| $5n+c$ | -1 | -2 | -3 | ... |

Answer: $2n^2 - n$

(2 marks)

b) How many dots will there be in pattern 40?

$2 \times 40^2 - 40 = 3160$

Answer: 3160

(2 marks)

c) Work out which pattern has 435 dots.

$2n^2 - n = 435$

$2n^2 - n - 435 = 0$

$n = 15, -14.5$ by quadratic formula / Completing square

(✓) (✗)

Answer: 15

(3 marks)

Q7. A sequence has n$^{th}$ term given by $n^2 - 6n$ . Two consecutive terms in the sequence have a difference of 75.
Find which two terms these are.

$(n+1)^2 - 6(n+1) - (n^2 - 6n) = 75$

$n^2 + 2n + 1 - 6n - 6 - n^2 + 6n = 75$

$2n = 80$

$n = 40.$

$\Rightarrow \quad 40^2 - 6(40) = 1360$

and $\quad 41^2 - 6(41) = 1435$

Answer: 1360, 1435

(4 marks)

Q8. Mark, an amateur mathematician, saves money each month. The amounts he saves follow the sequence 15p, 18p, 27p, 42p, 63p …

a) State how much Mark will save in the 6th month.

15, 18, 27, 42, 63, 90, 123

3, 9, 15, 21, 27, 33

6, 6, 6, 6, 6

Answer: 90, 123

(1 mark)

b) Work out how many months it will take before he is saving more than £50 a month.

$a = \frac{1}{2}(6)$

$\Rightarrow a = 3$

| $n$ | 1 | 2 | 3 | 4 | .. |
|---|---|---|---|---|---|
| $an^2$ | 3 | 12 | 27 | 48 | . |
| Target | 15 | 18 | 27 | 42 | ... |
| $bn+c$ | 12 | 6 | 0 | $-6$ | ... |

$\Rightarrow bn+c$ is $-6n+18$

$3n^2 - 6n + 18$ is nth term

Want $3n^2 - 6n + 18 > 5000$

$3n^2 - 6n - 4982 > 0$

$n > 41.7$, $n < -39.8$ (reject)

$\Rightarrow n = 42$.

Answer: 42

(5 marks)

Q9. Prove that every term of the sequence $n^2 - 10n + 40$ is positive.

$(n-5)^2 - 25 + 40$

$= (n-5)^2 + 15$

$\geq 0 \quad > 0$

$\Rightarrow n^2 - 10n + 40 > 0$

Answer: _____

(4 marks)

Q10. Work out which is the smallest term of the sequence $2n^2 - 8n + 13$

$2[n^2 - 4n] + 13$

$= 2[(n-2)^2 - 4] + 13$

$= 2(n-2)^2 - 8 + 13$

$= 2(n-2)^2 + 5$

min. at $(2, 5)$

$\Rightarrow n = 2$, term is $2(2^2) - 8(2) + 13$

$= 8 - 16 + 13$

$= 5$

Answer: 5

(4 marks)