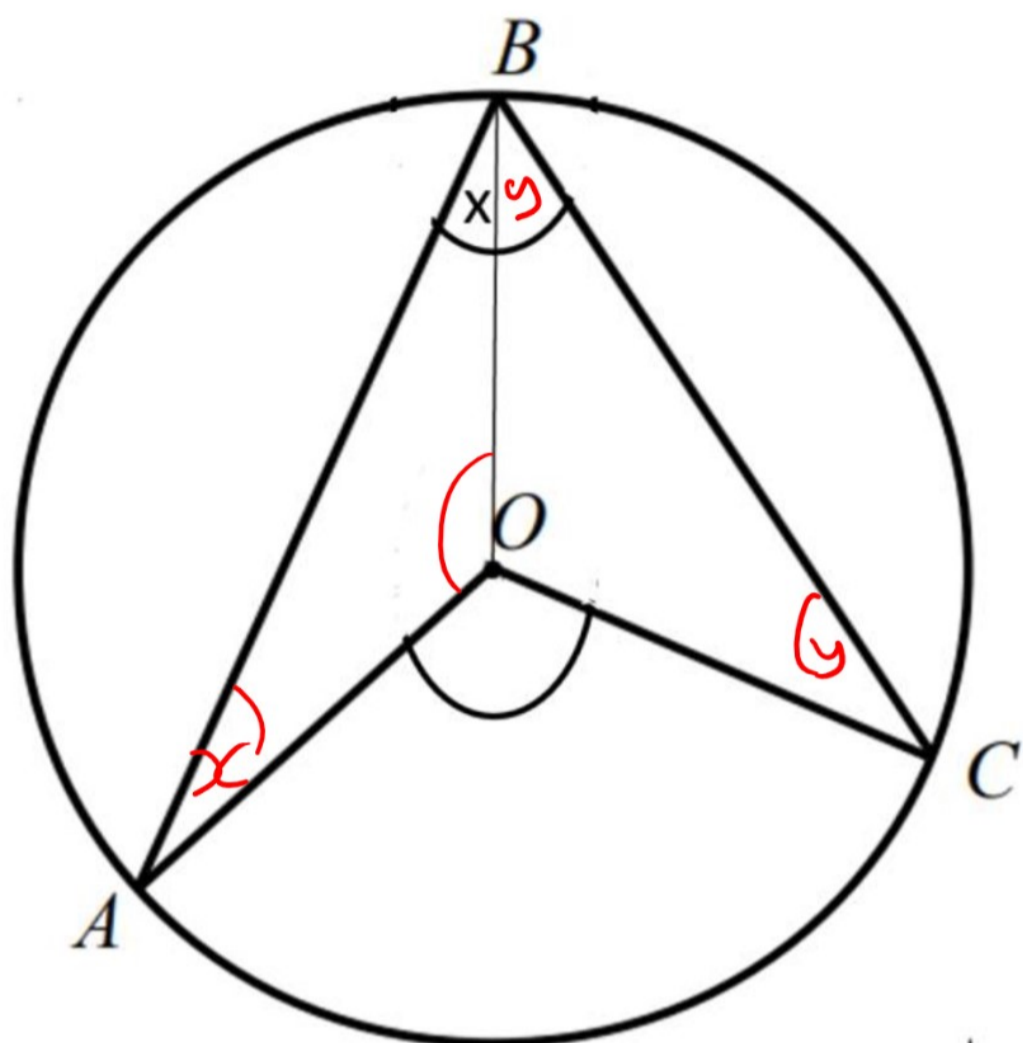




Proving Circle Theorem Exam Practice

Q1. A, B and C are points on the circumference of a circle, centre O.
Let x be angle ABO.



(a) Find an expression for AOB in terms of x .

$$AOB = 180^\circ - 2x$$

Answer: 180° - 2x
(2 marks)

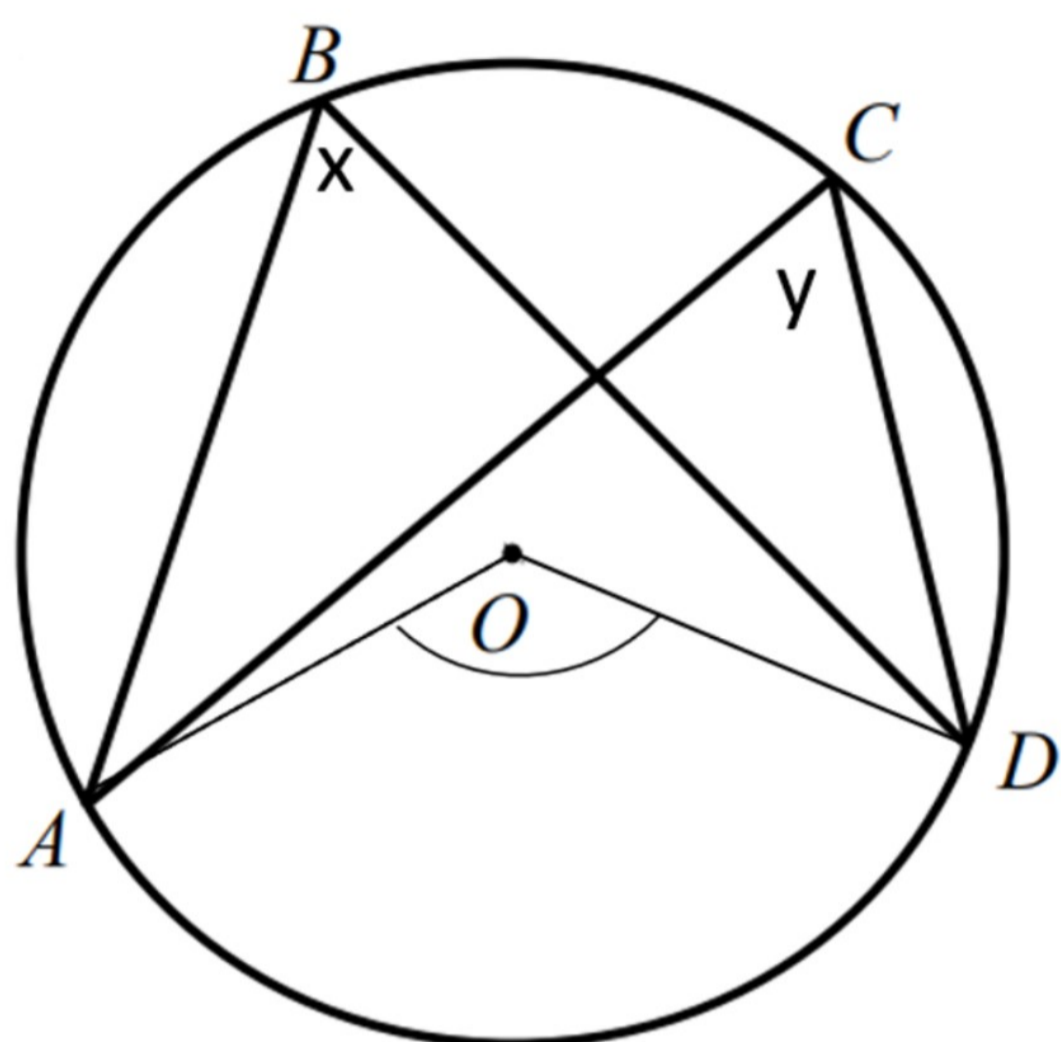
(b) Hence, prove that the angle AOC shown above is twice the size of angle ABC.

$$\begin{aligned}BOC &= 180 - 2y \\ \Rightarrow AOC &= 360 - (180 - 2x) - (180 - 2y) \\ &= 2x + 2y \\ &= 2(x + y) \\ &= 2\angle ABC \quad \square\end{aligned}$$

Answer: _____
(2 marks)



Q2. A, B, C and D are points on the circumference of a circle, centre O.



a) State the size of AOD in terms of x , justifying your reasoning.

$AOD = 2x$, using the theorem that "the angle at the centre is twice the angle at the circumference".

Answer: _____
(2 marks)

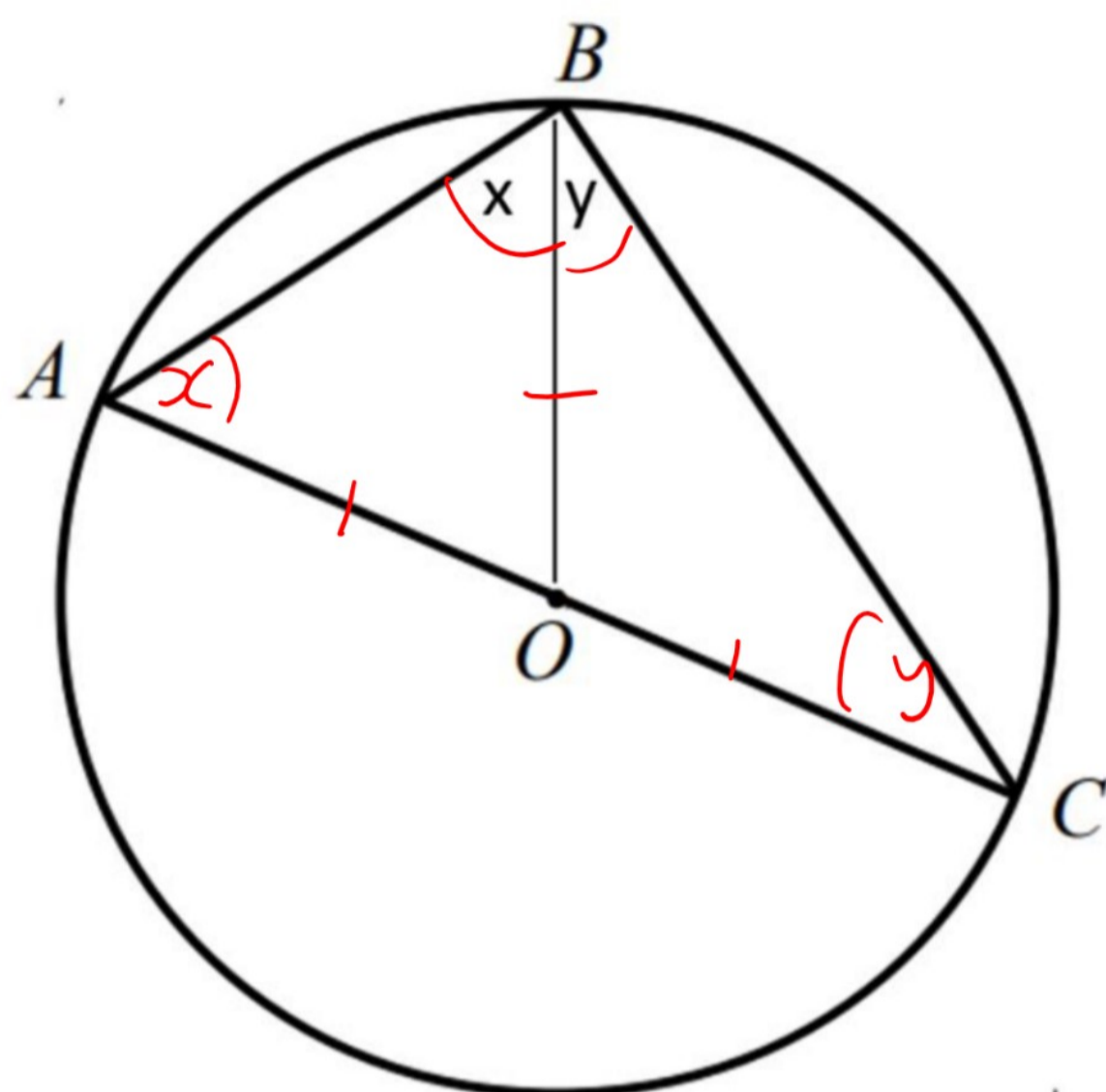
b) Hence prove that angle ABD and angle ACD are equal.

$AOD = 2y$ (angle at centre = $2 \times$ angle circumference)
 $\Rightarrow 2x = 2y$ from part (a)
 $\Rightarrow x = y$
 $\Rightarrow \angle ABD = \angle ACD \quad \square$

Answer: _____
(2 marks)



Q3. A, B and C are points on the circumference of a circle, centre O. AOC is a diameter of the circle. Let x be angle ABO and y be angle CYO.



a) Show that $2x + 2y = 180$, fully showing your reasoning.

• $\triangle BOC, \triangle AOB$ are isosceles triangles.

$$\angle AOB = 180 - 2x \text{ and } \angle BOC = 180 - 2y$$

• AOC is a straight line, so $180 - 2x + 180 - 2y = 180$
 $\Rightarrow 180 = 2x + 2y$

Answer: _____

(2 marks)

b) Hence prove that angle ABC is 90°

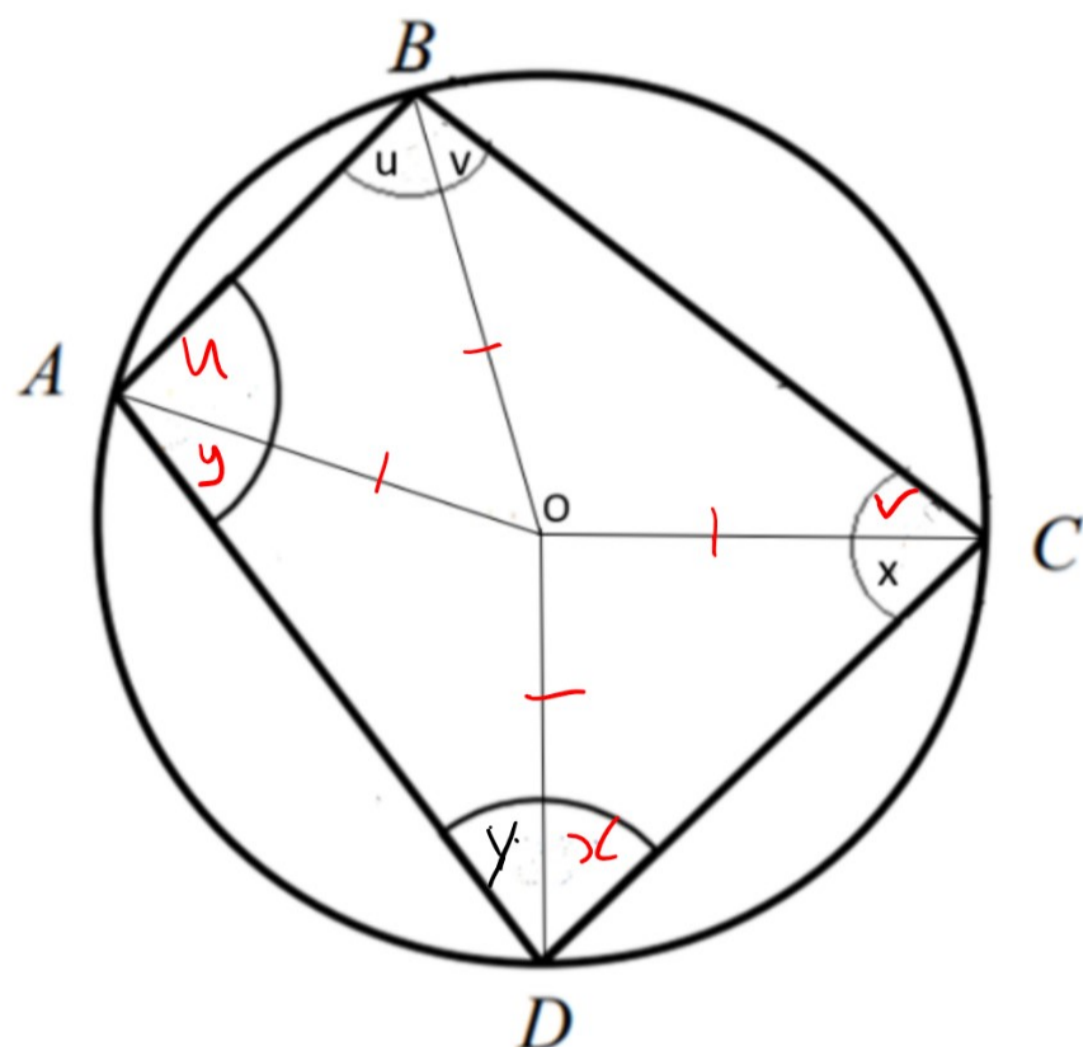
$$\begin{aligned} 2x + 2y &= 180 \\ \Rightarrow x + y &= 90^\circ \\ \Rightarrow \angle ABC &= 90^\circ \end{aligned}$$

Answer: _____

(2 marks)



Q4. A, B, C and D are points on the circumference of a circle, centre O.



a) Show that $x + y + u + v = 180$, fully showing your reasoning.

$$2x + 2y + 2u + 2v = 360 \quad (\text{angles of a quadrilateral total } 360^\circ)$$
$$\Rightarrow x + y + u + v = 180$$

Answer: _____
(2 marks)

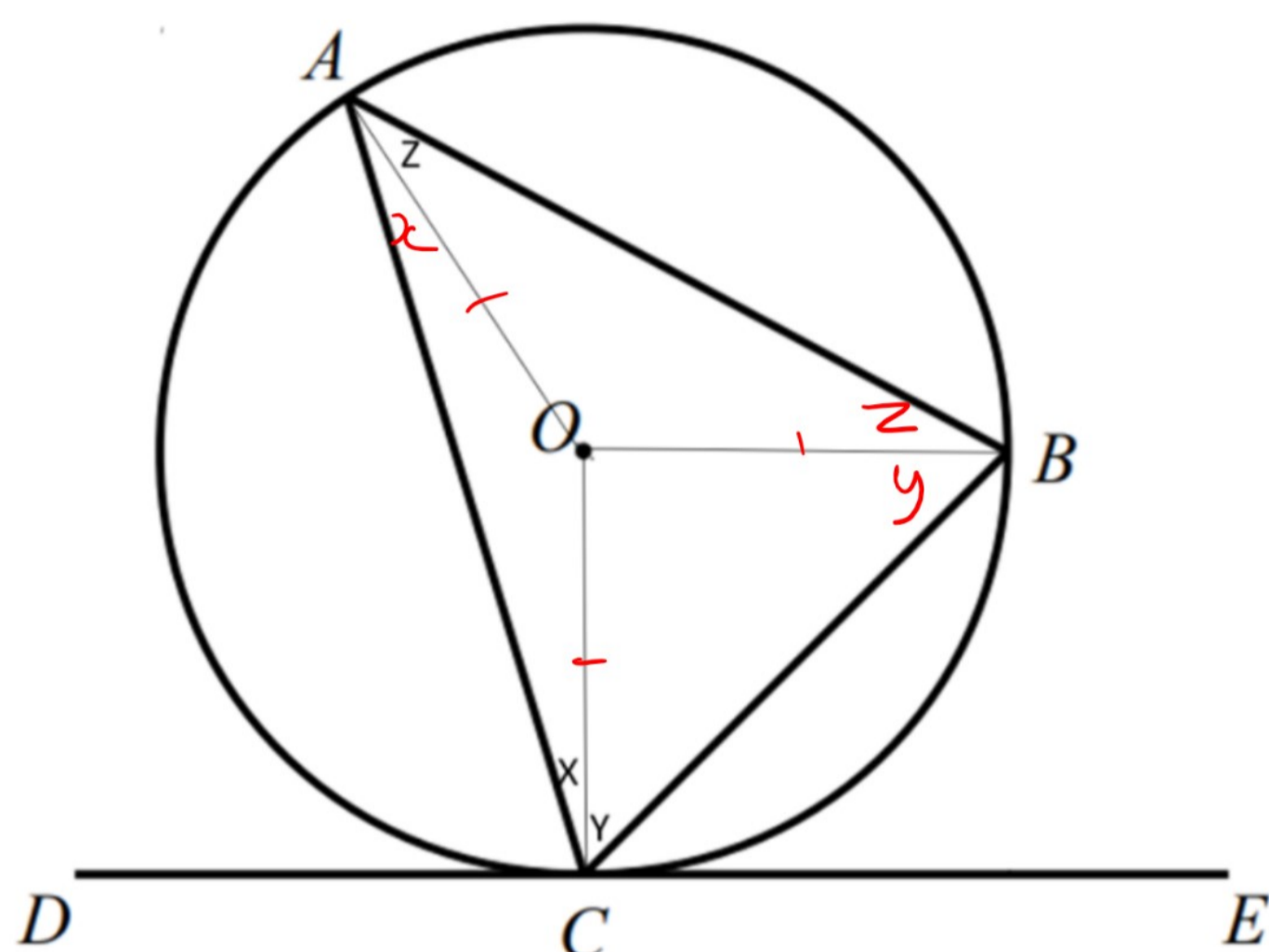
b) Hence prove that angle ABC and angle ADC add to 180°

$$ABC = u + v, \quad ADC = x + y$$
$$\text{from part (a), } ABC + ADC = 180^\circ$$

Answer: _____
(2 marks)



Q5. A, B and C are points on the circumference of a circle, centre O.
DCE is a tangent to the circle.



a) Show that $x + y + z = 180$, fully showing your reasoning.

$$180 - 2x + 180 - 2y + 180 - 2z = 360$$

$$180 = 2x + 2y + 2z$$

$$90 = x + y + z \quad \square$$

Answer: _____
(2 marks)

b) State the value of angle BCE in terms of y , justifying your answer.

$$\angle BCE = 90 - y \quad (\text{OC meets DE (tangent) at } 90^\circ)$$

Answer: _____
(2 marks)

c) Hence prove that angle BCE and angle BAC are equal.

$$(90 - y) \qquad (x + z)$$

$$\text{from (a), } x + y + z = 90$$

$$\Rightarrow x + z = 90 - y$$

$$\Rightarrow \angle BAC = \angle BCE \quad \square$$

Answer: _____
(2 marks)