Proof Exam Practice

Q1. Prove, using algebra, that the sum of any three consecutive integers is always divisible by three.

$n + (n+1) + (n+2)$

$= 3n + 3$

$= 3(n+1)$

$\therefore$ the sum of any three consecutive integers is divisible by 3

Answer: ____________________

(2 marks)

Q2. Prove, using algebra, that the sum of any two consecutive integers is always odd.

- $n + (n+1) = 2n+1$
- $2n+1$ represents any odd integer (if n integer)

$\therefore$ the sum of any two consecutive integers is odd.

Answer: ____________________

(2 marks)

Q3. Prove that the difference between the squares of any 2 consecutive integers is always odd.

$(n+1)^2 - n^2$

$= n^2 + 2n + 1 - n^2$

$= 2n + 1$

$\therefore$ the difference between the squares of any 2 consecutive integers is always odd

Answer:____________________

(2 marks)

Q4. Prove, using algebra, that the sum of any three odd integers is always odd.

$(2p+1) + (2q+1) + (2r+1)$ (where p, q, r are all distinct)

$= 2p + 2q + 2r + 3$

$= 2p + 2q + 2r + 2 + 1$

$= 2(p+q+r+1) + 1$

$\therefore$ the sum of any three odd integers is odd.

Answer:____________________

(2 marks)

Q5. Let n be an even number. Prove that n^2 is even.

- let $n = 2m$, where m is an integer.
- $(2m)^2 = 4m^2$

$= 2(2m^2)$

- $\therefore$ n^2 is even

Answer:______________________

(2 marks)

Q6. Prove that $(2n-5)^2 + n^2 + 25n + 1$ is never divisible by 5 for all whole values of n.

$4n^2 - 20n + 25 + n^2 + 25n + 1$

$= 5n^2 + 5n + 26$

$= 5n^2 + 5n + 25 + 1$

$= 5(n^2 + n + 5) + 1$

$\therefore$ it is never divisible by 5

Answer:______________________

(3 marks)

Q7. Prove that $m^2 + m$ is an even number for all values of m.

$= m(m+1)$

- let m be even. Then $m(m+1) =$ even $\times$ odd
  $=$ even
- let m be odd Then $m(m+1) =$ odd $\times$ even
  $=$ even

$\therefore m^2 + m$ is always even.

Answer:____________________

(3 marks)

Q8. Prove that the points D = (-5, 2), E = (-3, -4) and C = (3, -2) are the vertices of a isosceles triangle.

- we show that 2 sides are equal in length
- $DE = \sqrt{(-3--5)^2 + (-4-2)^2}$
  $= \sqrt{4+36}$
  $= \sqrt{40}$
- $CD = \sqrt{(-5-3)^2 + (2-2)^2}$
  $= \sqrt{64}$
  $= 8$
- $CE = \sqrt{(-3-3)^2 + (-4--2)^2}$
  $= \sqrt{40}$

$CE = DE \therefore$ isosceles

Answer:____________________

(3 marks)

Q9. Prove that $3n^2 - 4n + 10$ is positive for all values of n.

$$= 3\left(n^2 - \frac{4}{3}n\right) + 10$$

$$= 3\left(\left[n - \frac{4}{6}\right]^2 - \left(\frac{4}{6}\right)^2\right) + 10$$

$$= 3\left(n - \frac{2}{3}\right)^2 - \frac{12}{9} + 10$$

$$= \underbrace{3\left(n - \frac{2}{3}\right)^2}_{\geq 0} + \underbrace{\frac{26}{3}}_{>0}$$

$$> 0$$

Answer: ____________________

(4 marks)

Q10. Prove that $(4n + 3)^2 - (4n - 3)^2$ is always divisible by 6 for all whole values of n.

$$16n^2 + 24n + 9 - \left(16n^2 - 24n + 9\right)$$

$$= 24n$$

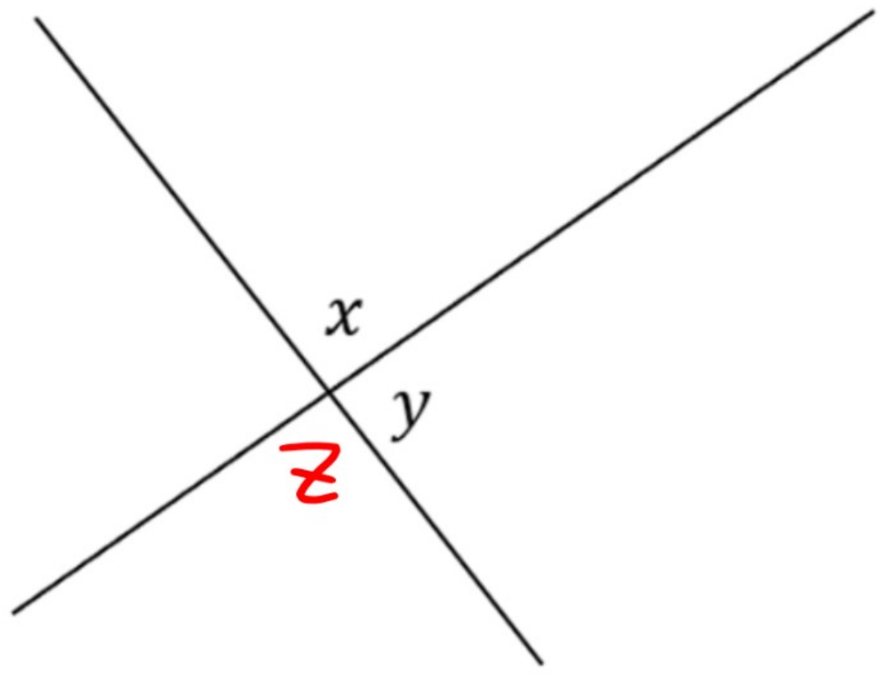
$$= 6(4n)$$

$\therefore$ it is always divisible by 6.

Answer: ____________________

(3 marks)

Q11.



(i) Find an expression for y in terms of x.

$y = 180 - x$

Answer: $y = 180 - x$

(1 mark)

(ii) Hence prove that "opposite angles at a point are equal".

- We show that $x = z$ (see diagram above)
- $y + z = 180$ (angles on a straight line)
- $180 - x + z = 180$

$\Rightarrow -x + z = 0$

$\Rightarrow x = z$ $\therefore$ opposite angles are equal.

Answer:

(2 marks)

Q12. Prove that the product of two surds is not always a surd.

- We show the counter-example :

$\sqrt{2} \times \sqrt{2} = 2$

$\therefore$ the product of two surds is not always a surd.

Answer:

(2 marks)

Q13. Using algebra, prove that $2^{48} - 1$ is not a prime number.

- $2^{48} - 1 = (2^{24}+1)(2^{24}-1)$
- Both $2^{24}+1$ and $2^{24}-1$ are not equal to 1 or $2^{48}-1$, so $2^{48}-1$ cannot be prime.

Answer:____________________

(3 marks)

Q14. Prove that $\sqrt{x} < x^2$ is not always true.

We show the counterexample $x = \frac{1}{4}$.

$\Rightarrow \sqrt{\frac{1}{4}} = \frac{1}{2}$, whilst $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\therefore \sqrt{x}$ is not always less than x^2

Answer:____________________

(2 marks)

Q15. A quadratic sequence has the formula, $an^2 + bn + c$ where n is a whole positive number, and a, b and c are whole numbers.

(i) Work out the first 3 terms of the quadratic sequence.

$n=1$ $\quad a+b+c$

$n=2$ $\quad 4a+2b+c$

$n=3$ $\quad 9a+3b+c$

Answer: $a+b+c$, $4a+2b+c$, $9a+3b+c$

(1 mark)

(ii) Hence prove that a is equal to half the *second difference* of the sequence

$a+b+c$, $\quad 4a+2b+c$, $\quad 9a+3b+c$

[1st differences] $\quad 3a+b \quad\quad 5a+b$

[2nd difference] $\quad 2a$

Answer:

(2 marks)

$\therefore a = \frac{1}{2}(2a)$, $\therefore a = \frac{1}{2}$ the 2nd difference

Q16. Prove that all square numbers are either a multiple of 4 or 1 more than a multiple of 4.

- even case : let $m = 2n$

$\Rightarrow m^2 = (2n)^2$
$= 4n^2$
$= 4(n^2)$ $\therefore$ multiple of 4

- odd case : let $m = 2n+1$

$m^2 = (2n+1)^2$
$= 4n^2+4n+1$
$= 4(n^2+n)+1$

$\therefore$ one more than a multiple of 4

Answer:

(4 marks)