

Equation of a Straight Line Exam Practice



Q1. Find an equation of the line which passes through the point P where $P = (0, 5)$ and is parallel to the line $y = -3x - 8$

- $y = mx + c$
- gradient of $y = -3x - 8$ is -3
- parallel means same gradient, so $m = -3$
- y-intercept will be 5 as $P = (0, 5)$ is on the line

$$\Rightarrow y = -3x + 5$$

Answer: $y = -3x + 5$
(2 marks)

Q2. Find an equation of the line which passes through the point P where $P = (0, -3)$ and is perpendicular to the line $y = 3x + 2$

- gradient of $y = 3x + 2$ is 3
- line perpendicular to this one is $-\frac{1}{3}$
- y-intercept is -3 , using $P = (0, -3)$ on the line

$$\Rightarrow y = -\frac{1}{3}x - 3$$

Answer: $y = -\frac{1}{3}x - 3$
(2 marks)



Q3. Find an equation of the line which passes through the point (2, -1) and is parallel to the line $y = \frac{1}{2}x - 4$

• $y = mx + c$.

• $m = \frac{1}{2}$

• substitute $(2, -1)$ and value for m to find c :

$$-1 = \frac{1}{2}(2) + c$$

$$-1 = 1 + c$$

$$-2 = c$$

$$\Rightarrow y = \frac{1}{2}x - 2$$

Answer: $y = \frac{1}{2}x - 2$
(2 marks)

Q4. Find an equation of the line which passes through the point (0, -3) and is perpendicular to the line $y = -\frac{3}{4}x + 2$

• $y = mx + c$

• line perpendicular to line with gradient $-\frac{3}{4}$ is $\frac{4}{3}$

• y-intercept is -3 as (0, -3) is on the line

$$\Rightarrow y = \frac{4}{3}x - 3$$

Answer: $y = \frac{4}{3}x - 3$
(2 marks)



Q5. Line L passes through the points $A = (2, 5)$, $B = (5, 17)$. Find the equation of the line M which is perpendicular to line L and passes through point B .

$$\begin{aligned} \cdot \text{gradient line segment } AB &= \frac{17-5}{5-2} \\ &= \frac{12}{3} \\ &= 4. \end{aligned}$$

$$\cdot \text{gradient line } M = -\frac{1}{4}$$

$$\cdot y = -\frac{1}{4}x + c; \text{ substitute } (5, 17) \text{ to find } c:$$

$$17 = -\frac{1}{4}(5) + c$$

$$\frac{73}{4} = c$$

$$\text{Answer: } y = -\frac{1}{4}x + \frac{73}{4}$$

(3 marks)

Q6. Line M passes through the points $A = (6, 5)$, $B = (10, 3)$. Find the equation of the line N which is parallel to line M and passes through the point $(-7, 4)$.

$$\begin{aligned} \cdot \text{gradient line } M &= \frac{3-5}{10-6} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2} \quad (= \text{gradient } N \text{ also}) \end{aligned}$$

$$\cdot y = -\frac{1}{2}x + c; \text{ substitute } (-7, 4) \text{ to find } c$$

$$4 = -\frac{1}{2}(-7) + c$$

$$4 = \frac{7}{2} + c$$

$$\frac{1}{2} = c$$

$$\text{Answer: } y = -\frac{1}{2}x + \frac{1}{2}$$

(3 marks)



Q7. Line L has equation $y = 2 - \frac{1}{4}x$ and line M has equation $y = 4x - 6$.

Show that the two lines are perpendicular to each other.

- $L: y = -\frac{1}{4}x + 2 \Rightarrow \text{gradient} = -\frac{1}{4}$
- $M: y = 4x - 6 \Rightarrow \text{gradient} = 4$
- $4 \times -\frac{1}{4} = -1$ so L and M are perpendicular.

Answer: _____

(2 marks)

Q8. Line L has equation $8x - 4y - 3 = 0$ and line M has equation $2y - 9 = x$. Decide if L and M are perpendicular to each other. You must show all your working.

- $L: 8x - 4y - 3 = 0$
 $\Rightarrow 8x - 3 = 4y$
 $\Rightarrow 2x - \frac{3}{4} = y$
so gradient L is 2 .

- $M: 2y - 9 = x$
 $\Rightarrow 2y = x + 9$
 $\Rightarrow y = \frac{1}{2}x + \frac{9}{2}$
so gradient M is $\frac{1}{2}$

- $2 \times \frac{1}{2} \neq -1 \Rightarrow L, M$ not perpendicular

Answer: No

(3 marks)



Q9. Decide which two of these lines are parallel to each other:

1. $3x + 9 = 12y$

$$\Rightarrow \frac{3}{12}x + \frac{9}{12} = y$$

2. $y = 5$

$$\Rightarrow y = 0x + 5$$

3. $4y - 16x + 9 = 0$

$$\Rightarrow 4y = 16x - 9 \Rightarrow y = 4x - \frac{9}{4}$$

4. $8y - 2x = 10$

$$\Rightarrow 8y = 2x + 10 \Rightarrow y = \frac{2}{8}x + \frac{10}{8}$$

5. $0 = 15 - 4y - 2x$

$$\Rightarrow 4y = 15 - 2x \Rightarrow y = -\frac{2}{4}x + \frac{15}{4}$$

So (1) gradient $\frac{3}{12}$ $(\frac{1}{4})$

(2) gradient 0

(3) gradient 4

(4) gradient $\frac{2}{8}$ $(\frac{1}{4})$

(5) gradient $-\frac{2}{4}$ $(-\frac{1}{2})$

Answer: 1, 4 are parallel
(2 marks)

Q10. The line which passes through (a, 3) and (5, 11) is parallel to the line which passes through (4, 7) and (14, 12). Find the value of a.

line through (4, 7) and (14, 12) is $\frac{12-7}{14-4}$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

line through (a, 3) and (5, 11) is

$$\frac{11-3}{5-a}$$

$$= \frac{8}{5-a}$$

$$\frac{8}{5-a} = \frac{1}{2} \Rightarrow 16 = 5-a$$

$$\Rightarrow a = -11$$

Answer: -11

(3 marks)



Q11. State the equation of a line which is perpendicular to the line $y = 2$

- $x = 5$
- (any vertical line of the form $x = a$ for any number a is correct here)

Answer: $x = 5$
(1 mark)

Q12. Decide which two of these lines are perpendicular to each other:

1. $y - 3x + 9 = 0 \Rightarrow y = 3x - 9$
2. $3x - 5y + 1 = 0 \Rightarrow 5y = 3x + 1 \Rightarrow y = \frac{3}{5}x + \frac{1}{5}$
3. $-2x + y = 11 \Rightarrow y = 2x + 11$
4. $4y - 2x + 9 = 0 \Rightarrow 4y = 2x - 9 \Rightarrow y = \frac{2}{4}x - \frac{9}{4}$
5. $3y = 19 - 5x \Rightarrow y = -\frac{5}{3}x + \frac{19}{3}$

① has gradient 3

② has gradient $\frac{3}{5}$

③ has gradient 2

④ has gradient $\frac{2}{4}$ ($\frac{1}{2}$)

⑤ has gradient $-\frac{5}{3}$

\Rightarrow ② and ⑤ are perpendicular as $\frac{3}{5} \times -\frac{5}{3} = -1$

Answer: ②, ⑤
(3 marks)



Q13. The line which passes through $(a, -4)$ and $(9, -6)$ is perpendicular to the line which passes through $(12, 19)$ and $(15, 13)$. Find the value of a .

• line through $(12, 19), (15, 13)$ has grad. $\frac{13-19}{15-12}$

$$= \frac{-6}{3}$$

$$= -2$$

• line through $(a, -4), (9, -6)$ has grad $\frac{-6--4}{9-a}$

$$= \frac{-2}{9-a}$$

• We need $-2 \times \frac{-2}{9-a} = -1$ as the two lines are perpendicular

$$\Rightarrow \frac{4}{9-a} = -1$$

$$\Rightarrow 4 = -(9-a)$$

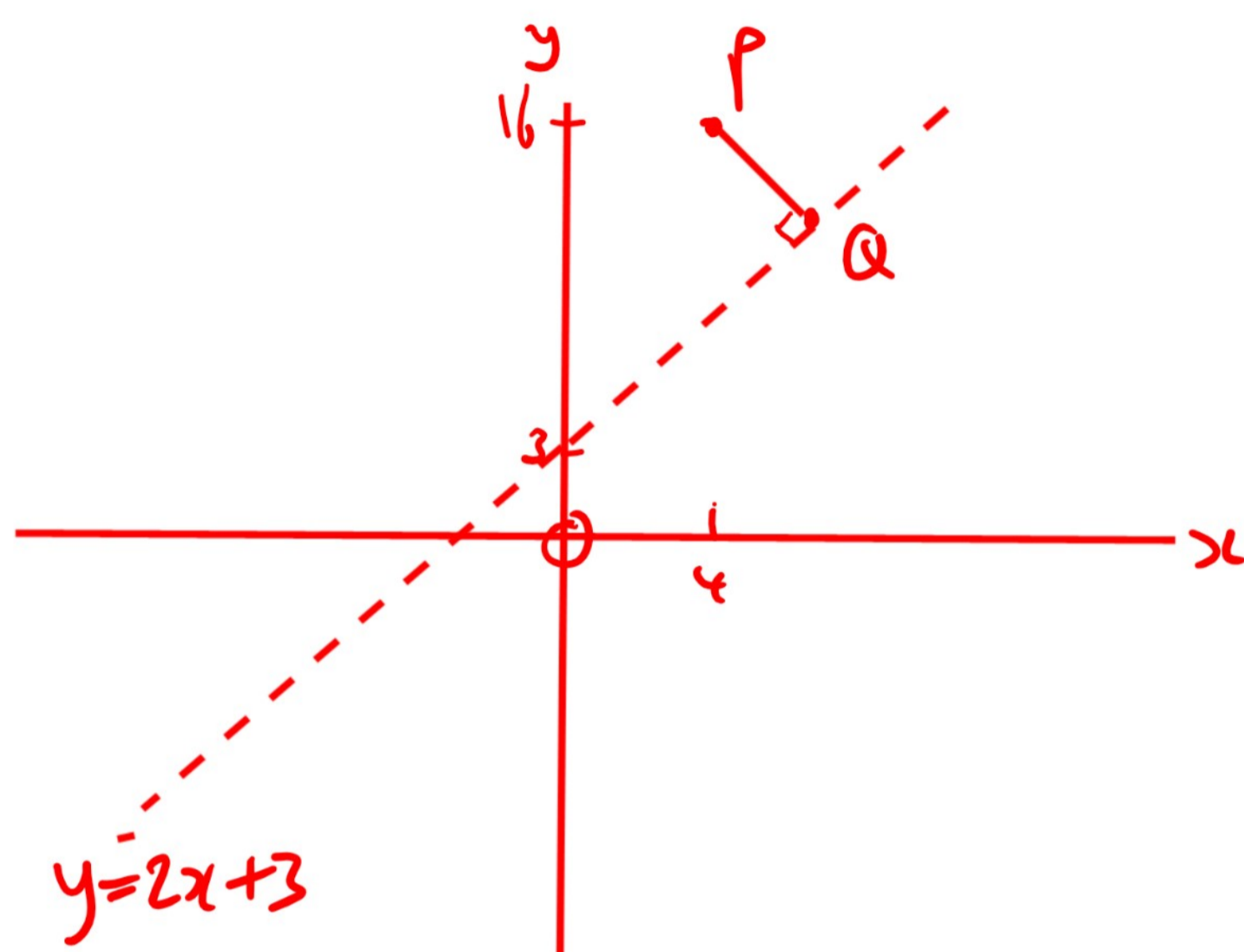
$$\Rightarrow 4 = -9 + a$$

$$\Rightarrow 13 = a$$

Answer: $a = 13$
(3 marks)



Q14. A line has equation $y = 2x + 3$. Let P be the point (4,16) and point Q be the closest point to P which lies on the line. Find the co-ordinates of Q.



Closest point. \Rightarrow
PQ is perpendicular
to $y = 2x + 3$

- Need to find the line through PQ and then solve simultaneously with $y = 2x + 3$ to find Q.
- Gradient $y = 2x + 3$ is 2, so gradient PQ is $-\frac{1}{2}$
- $y = -\frac{1}{2}x + c$; substitute P to find c:
 $16 = -\frac{1}{2}(4) + c$
 $16 = -2 + c$
 $c = 18$
- $$\left. \begin{array}{l} y = -\frac{1}{2}x + 18 \\ y = 2x + 3 \end{array} \right\} 2x + 3 = -\frac{1}{2}x + 18$$
- $\frac{5}{2}x = 15 \Rightarrow x = 6$; sub in $y = 2x + 3$
 $y = 2(6) + 3 \Rightarrow y = 15$
 $Q = (6, 15)$

Answer: (6, 15)

(6 marks)