Inverse and Composite Functions Exam Practice

Q1. Here is a function: $f(x) = 5x - 9$

a) Work out the value of $f(-4)$

$f(-4) = 5(-4) - 9$

$= -29$

Answer: -29

(1 mark)

b) Work out the value of $f^{-1}(46)$

$f^{-1}(x) = \frac{x+9}{5}$

$f^{-1}(46) = \frac{46+9}{5}$

$= 11$

Answer: 11

(2 marks)

Q2. Here is a function: $g(x) = \sqrt{x} + 3$

a) Work out the value of $g(144)$

$g(144) = \sqrt{144} + 3$

$= 12 + 3$

$= 15$

Answer: 15

(1 mark)

b) What is the input when the output value of the function is 72 ?

$\sqrt{x} + 3 = 72$

$\sqrt{x} = 69$

$x = 69^2$

$x = 4761$

Answer: 4761

(2 marks)

Q3. Let f and g be the functions: $f(x) = x^2 - 3$ and $g(x) = x + 7$

a) Work out the value of $fg(-2)$

$g(-2) = -2+7$
$= 5$

$\Rightarrow f(g(-2)) = f(5)$
$= 5^2-3$
$= 22$

Answer: 22

(1 mark)

b) Work out the value of $gf(6)$

$f(6) = 6^2-3$
$= 33$

$\Rightarrow g(f(6)) = g(33)$
$= 33+7$
$= 40$

Answer: 40

(1 mark)

c) Find the expression corresponding to $gf(x)$

$gf(x) = g(x^2-3)$
$= (x^2-3)+7$
$= x^2+4$

Answer: x^2+4

(2 marks)

Q4. A function is defined by $g(x) = ax + b$ where a and b are numbers to be found. Given that $g(3) = 10$ and $g(8) = 12$, find the value of a and b.

$g(3)=10 \Rightarrow 3a+b=10 \quad (1)$

$g(8)=12 \Rightarrow 8a+b=12 \quad (2)$

$(2)-(1) \Rightarrow 5a=2$

$a=\frac{2}{5}$, subs in (1):

$3(\frac{2}{5})+b=10$

$\Rightarrow \frac{6}{5}+b=10$

$b=\frac{44}{5}$

Answer: $a=\frac{2}{5}, b=\frac{44}{5}$

(3 marks)

Q5. Let $f(x)$, $g(x)$ be defined by $f(x) = 3x + 2$ and $g(x) = x^2 + 7$ such that $fg(a) = 71$. Find the possible values of a.

$f(g(a))=71 \Rightarrow f(a^2+7)=71$

$\Rightarrow 3(a^2+7)+2=71$

$\Rightarrow 3a^2+21+2=71$

$\Rightarrow 3a^2=48$

$\Rightarrow a^2=16$

$a=4, -4$

Answer: $a=\pm 4$

(3 marks)

Q6. Let $f(x)$, $g(x)$ be defined by $f(x) = x^2$ and $g(x) = 3x + 2$ such that

a) Find an expression for $fg(x)$.

$f(g(x)) = f(3x+2)$

$= (3x+2)^2$

$(= 9x^2 + 12x + 4)$

Answer: $(3x+2)^2$

(2 marks)

b) Solve $fg(x) = g(f(x))$, leaving your answer in surd form.

$g(f(x)) = g(x^2)$

$= 3x^2 + 2$

Solve $(3x+2)^2 = 3x^2 + 2$

$\Rightarrow \quad 9x^2 + 12x + 4 = 3x^2 + 2$

$\Rightarrow \quad 6x^2 + 12x + 4 = 0$

$\Rightarrow \quad 3x^2 + 6x + 2 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 4(3)(2)}}{2(3)}$$

$$= \frac{-6 \pm \sqrt{12}}{6}$$

$= -1 \pm 2\sqrt{3}$

Answer: $-1 \pm 2\sqrt{3}$

(4 marks)

Q7. Let $f(x)$ be defined by $f(x) = \frac{1}{x+1}$ together with the restriction $x \neq -1$

a) Show that $ff(x) = \frac{x+a}{x+b}$, where a and b are numbers to be found.

$$ff(x) = f\left(\frac{1}{x+1}\right)$$

$$= \frac{1}{\frac{1}{x+1} + 1}$$

$$= \frac{1}{\frac{1+x+1}{x+1}}$$

$$= \frac{1}{\frac{2+x}{x+1}}$$

$$= \frac{x+1}{2+x}$$

Answer: $\frac{x+1}{x+2}$

(4 marks)

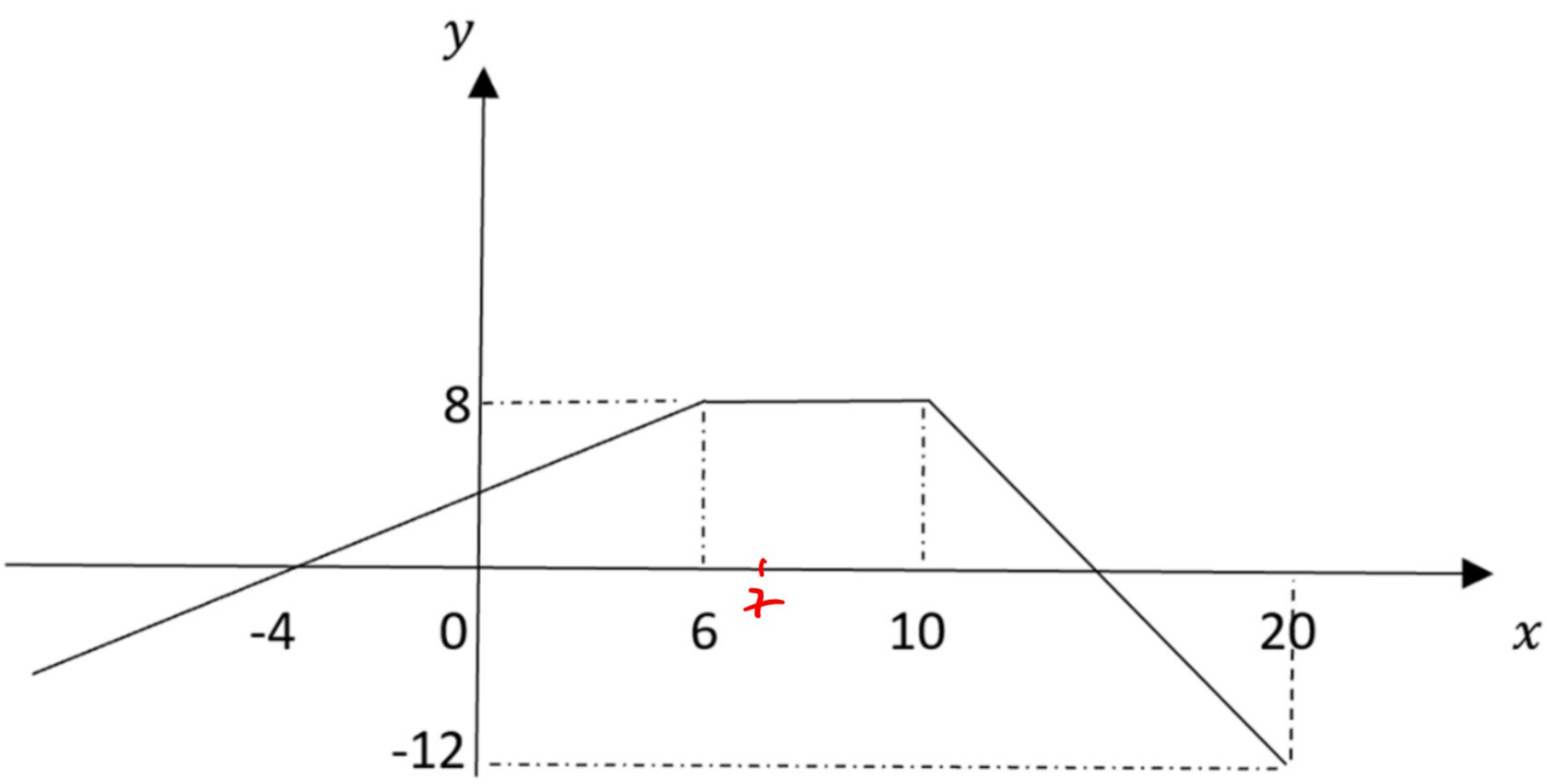
b) State the necessary restriction on the input values to $ff(x)$.

$x \neq -2$

Answer: $x \neq -2$

(1 mark)

Q8. A sketch of a function $f(x)$, composed of three lines, is shown below.



a) State the value of $f(7)$

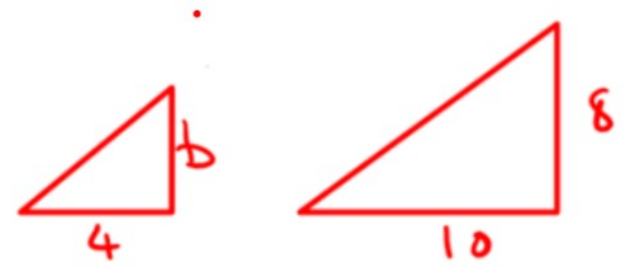
$f(7) = 8$ from the graph

Answer: 8

(1 mark)

b) Work out the value of $ff(-4)$, giving your answer as a fraction in its simplified form.

- $f(-4) = 0$, and $f(f(-4)) = f(0)$
- Let $b = f(0)$: Using similar triangles,



$\frac{b}{4} = \frac{8}{10} \Rightarrow 10b = 32$

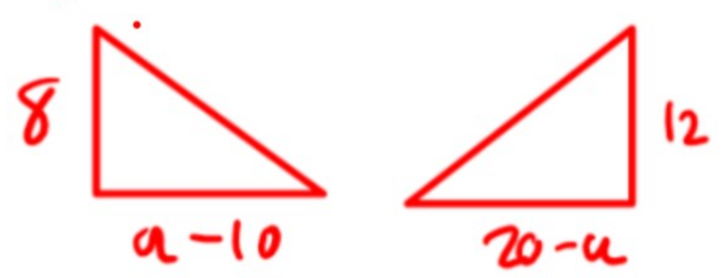
$b = \frac{32}{10}$

Answer: $\frac{16}{5}$

(1 mark)

c) Find the values of $f^{-1}(0)$

- $x = -4$ from the graph directly.
- let a be the other solution. Using similar triangles :



$\frac{20-a}{a-10} = \frac{12}{8}$

$160 - 8a = 12a - 120$

$280 = 20a \Rightarrow a = 14$

Answer: -4, 14

(2 marks)

Applied Mixed Practice Problems

Q9. A scientist models the volume (cm^3) of gas G produced by a chemical reaction over time T after the start of the experiment (in seconds). She does this using the formula,

$$G = 35\sqrt{T - 50} + 100, \quad \text{where } T \geq 50 \text{ seconds}$$

(i) Explain why the condition $T \geq 50$ is necessary.

For $\sqrt{T-50}$ to be defined, $T - 50 \geq 0$

$\Rightarrow T \geq 50$.

Answer:____________________

(1 mark)

(ii) Use the model to predict the volume of gas after 2¼ minutes, giving your answer to the nearest cm^3.

$2\frac{1}{4}$ mins $= 2 \times 60 + 15$ seconds

$= 135$ seconds

$G = 35\sqrt{135 - 50} + 100$

$\Rightarrow G = 422.68$

Answer: $423 cm^3$

(2 marks)

(iii) Find the time when there will be 0.15 litres of gas, giving your answer to the nearest second. $(= 150 cm^3)$

$150 = 35\sqrt{T - 50} + 100$

$\frac{150 - 100}{35} = \sqrt{T - 50}$

$\left(\frac{150 - 100}{35}\right)^2 + 50 = T$

$T = 52.04$ s

Answer: 52 seconds

(4 marks)