Direct and Inverse Proportion Exam Practice

Q1. a is directly proportional to b. When a = 6, b = 24, find the value of b when a = 7.

- $a = kb$
- $\Rightarrow 6 = k \times 24$
- $\Rightarrow k = \dfrac{6}{24}$
- $k = \dfrac{1}{4}$

- $7 = \dfrac{1}{4}(b)$
- $\Rightarrow b = 7 \times 4$
- $b = 28$

Answer:___28___

(3 marks)

Q2. c is inversely proportional to the square of d. When c = 18, d = 2, find the value of d when c = 2, given that d is positive.

- $C = \dfrac{k}{d^2}$

- $18 = \dfrac{k}{4}$

- $\Rightarrow k = 18 \times 4 = 72$

- $2 = \dfrac{72}{d^2}$

- $\Rightarrow 2d^2 = 72$
- $d^2 = 36 \Rightarrow d = \sqrt{36}$

Answer:___6___

(3 marks)

GCSEMathsRevision.com

Q3. x is directly proportional to the square root of y. When x = 4, y = 64, find the value of x when y = 144.

$$x = k\sqrt{y}$$

$$4 = k\sqrt{64}$$

$$\Rightarrow 4 = k \times 8$$

$$k = \frac{4}{8}$$

$$= \frac{1}{2}$$

$$x = \frac{1}{2}\sqrt{144}$$

$$= \frac{1}{2}(12)$$

$$= 6$$

Answer: _____6_____

(3 marks)

Q4. a is inversely proportional to b. When a = 5, b = 12, find the value of b when a = 7.5

$$a = \frac{k}{b}$$

$$5 = \frac{k}{12}$$

$$\Rightarrow k = 60$$

$$7.5 = \frac{60}{b}$$

$$b = \frac{60}{7.5} \Rightarrow b = 8$$

Answer: _____8_____

(3 marks)

Q5. a) Here is a table of values showing the corresponding values of x and y:

x	3	4
y	54	128

State which of these relationships holds between x and y?
A) $y \propto x$ B) $y \propto \sqrt{x}$ C) $y \propto x^3$ D) $y \propto x^2$

• By inspection, A, B are not possible. Try C):

$$54 = k(3^3) \qquad 128 = k(4^3)$$

$$k = 2, \qquad k = 2 \qquad \Rightarrow \text{Same } k \text{ value, so C works}$$

Answer: __C__

(1 mark)

b) Find an explicit formula for y in terms of x.

By (a), $k = 2$ so $y = 2x^3$

Answer: __$y = 2x^3$__

(2 marks)

GCSEMathsRevision.com

Q6. a) The variable y is inversely proportional to the square root of x. Use the table of values below to find an explicit formula for y in terms of x.

x	4	9
y	24	16

- $y = \dfrac{k}{\sqrt{x}}$

- $24 = \dfrac{k}{\sqrt{4}}$

$24 \times 2 = k$

$48 = k$

$y = \dfrac{48}{\sqrt{x}}$

Answer: $y = \dfrac{48}{\sqrt{x}}$

(2 marks)

b) If y is equal to 144, find the value of x.

$144 = \dfrac{48}{\sqrt{x}}$

$\sqrt{x} = \dfrac{48}{144}$

$\sqrt{x} = \dfrac{1}{3}$

$x = \left(\dfrac{1}{3}\right)^2$

$x = \dfrac{1}{9}$

Answer: $\dfrac{1}{9}$

(2 marks)

Problem Questions:

Q7. The mass in grams of a 3d shape is directly proportional to the cube of its height in cm. The surface area of the same shape is directly proportional to the square of its height. When the shape has height 8 cm, the mass is 1048 g, and the surface area is 32 cm^2.
Find the surface area, when the mass is 32 kg.

· Let M = mass (g)
x = hight (cm)
A = surface Area (cm^2)

· $M = k_1 x^3$ $A = k_2 x^2$

$1048 = k_1 (8^3)$ $32 = k_2 (8^2)$

$k_1 = \dfrac{131}{64}$ $k_2 = \dfrac{32}{64}$

$k_2 = \dfrac{1}{2}$

· $M = \dfrac{131}{64} x^3$

$32000 = \dfrac{131}{64} x^3$

$\sqrt[3]{\dfrac{64 \times 32000}{131}} = x$

$x = 25.00$

$\Rightarrow A = \dfrac{1}{2}(25.00)^2$

$A = 312.61\ldots$

Answer: _____312.6 cm^2_____

(5 marks)

Q8. The frequency f of sound is inversely proportional to the wavelength w. A sound with a frequency of 32 hertz has a wavelength of 18 metres. Work out the frequency, when the frequency and the wavelength have the same numerical value.

$$f = \frac{k}{w}$$

$$32 = \frac{k}{18}$$

$$k = 576$$

$$\Rightarrow f = \frac{576}{w} \qquad \text{let } w = f$$

$$\Rightarrow f = \frac{576}{f}$$

$$\Rightarrow f^2 = 576$$

$$f = \sqrt{576}$$

$$= 24$$

Answer: <u>   24 hertz   </u>

(5 marks)

Q9. Three quantities x, y and z are such that x is inversely proportional to the square root of z, and y is proportional to x^3. In each case the constant of proportionality is the same value.
Complete the table below, showing all your steps of working out.

x	4	0.25
y	80	10
z	$\frac{5}{256}$	49

$$y = kx^3$$

$$80 = k(4^3)$$

$$\frac{80}{64} = k$$

$$k = 1.25$$

So $\boxed{y = 1.25x^3}$

$x = \frac{k}{\sqrt{z}}$, and using $k = 1.25$,

$$x = \frac{1.25}{\sqrt{49}}$$

$$x = \frac{1.25}{7}$$

$$x = 0.25$$

$$\Rightarrow y = 1.25(0.25^3)$$

$$y = \frac{5}{256}$$

Answer:_____

(4 marks)

GCSEMathsRevision.com

Q10. The number of minutes required to complete a task is inversely proportional to the square of the number of people working on it. If it takes 30 minutes to complete the task when 10 people are working on it, work out the minimum number of staff required to ensure that the task does not take longer than 1¼ hours.

- let: T = no. mins to complete task
  N = no. people working

- $T \sim \dfrac{k}{N^2}$

$30 = \dfrac{k}{10^2}$

$\Rightarrow k = 3000$

$\Rightarrow \boxed{T = \dfrac{3000}{N^2}}$

- $1\frac{1}{4}$ hrs = 75 mins

$75 = \dfrac{3000}{N^2}$

$N^2 = \dfrac{3000}{75}$

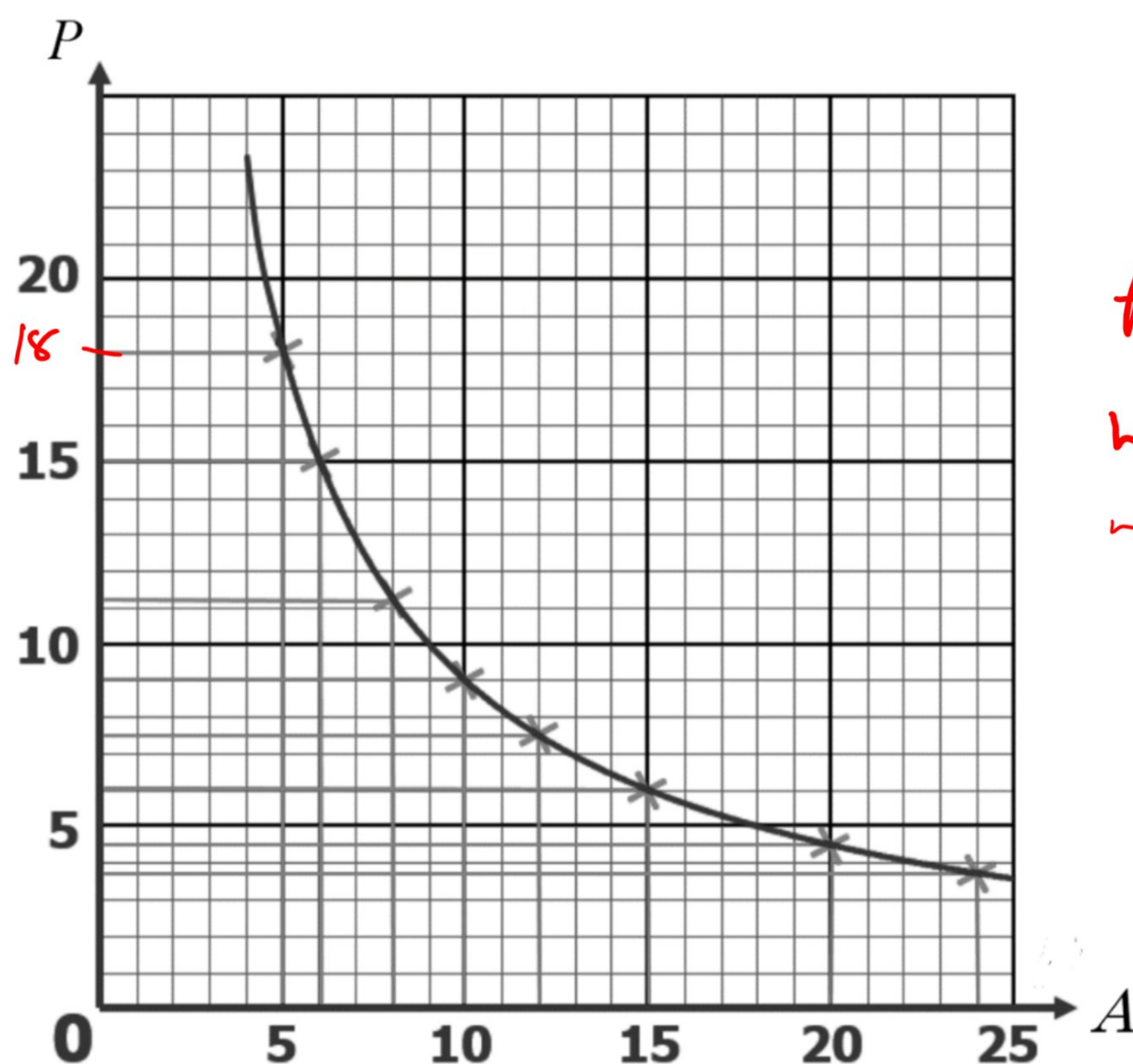$N^2 = 40$

$N = \sqrt{40} \approx 6.3$

$\Rightarrow$ need at least 7 people

Answer: ___7___

(4 marks)

Q11. Two physical quantities, A and P are measured in a lab and the results are plotted on the graph shown below:

from the graph:

when $A = 5$, $P = 18$
when $A = 10$, $P = 9$

(a) Determine the value of P if A is equal to 48.

- Try $P = \frac{k}{A}$ using these values

$18 = \frac{k}{5}$ and $9 = \frac{k}{10}$

$\Rightarrow k = 90$ $k = 90$

So it is indeed $P = \frac{90}{A}$ (as opposed to, e.g. $P = \frac{k}{A^2}$)

- Using $P = \frac{90}{A}$, when A=48,

$\Rightarrow P = \frac{90}{48}$

Answer: $\frac{15}{8}$ or 1.875

(3 marks)

(b) Determine the value of A if P = 2.

$P = \frac{90}{A}$

$2 = \frac{90}{A}$

$A = \frac{90}{2}$

$A = 45$

Answer: 45

(2 marks)

GCSEMathsRevision.com