



Recurring Decimal to Fraction Exam Practice

Q1. Convert $\frac{2}{15}$ to a decimal.

0.1 $\dot{3}$

Answer: 0.1 $\dot{3}$
(2 marks)

Q2. Convert $\frac{5}{11}$ to a decimal.

0.4 $\overline{5}$ or 0.4 $\dot{5}$

Answer: 0.4 $\overline{5}$
(2 marks)

Q3. Convert $\frac{4}{9}$ to a decimal.

0.4 $\overline{4}$

Answer: 0.4 $\overline{4}$
(2 marks)



Q4. Prove algebraically that the recurring decimal $0.363636\dots$ is equivalent to the fraction $\frac{4}{11}$

$$\begin{aligned} \text{let: } x &= 0.3636\dots \\ \Rightarrow 100x &= 36.3636\dots \\ \Rightarrow 99x &= 36 \\ x &= \frac{36}{99} \\ x &= \frac{4}{11} \end{aligned}$$

Answer: _____
(2 marks)

Q5. Prove algebraically that the recurring decimal $0.\dot{7}$ is equivalent to the fraction $\frac{7}{9}$

$$\begin{aligned} \text{let } x &= 0.777\dots \\ \Rightarrow 10x &= 7.777\dots \\ \Rightarrow 9x &= 7 \\ x &= \frac{7}{9} \end{aligned}$$

Answer: _____
(2 marks)



Q6. Prove algebraically that the recurring decimal $0.\dot{4}7$ is equivalent to the fraction $\frac{47}{99}$

$$\text{let } x = 0.4747 \dots -$$

$$100x = 47.4747 \dots -$$

$$\Rightarrow 99x = 47$$

$$x = \frac{47}{99}$$

Answer: _____

(2 marks)

Q7. Prove algebraically that the recurring decimal $0.0\dot{1}6$ is equivalent to the fraction $\frac{8}{495}$

$$\text{let } x = 0.0161616 \dots -$$

$$\Rightarrow 1000x = 16.1616 \dots$$

$$\text{and } 10x = 0.1616 \dots$$

$$\text{So } 990x = 16$$

$$x = \frac{16}{990}$$

$$x = \frac{8}{495}$$

Answer: _____

(2 marks)



Q8. Prove algebraically that the recurring decimal $0.1\dot{5}8$ is equivalent to the fraction $\frac{a}{b}$ where a, b are whole numbers to be found.

$$\text{Let } x = 0.15858585\dots$$

$$\Rightarrow 10000x = 1585.8585\dots$$

$$100x = 15.8585\dots$$

$$9900x = 1570$$

$$x = \frac{1570}{9900}$$

$$= \frac{157}{990}$$

Answer: $\frac{157}{990}$
(2 marks)

Q9. Prove algebraically that the recurring decimal $0.4\dot{2}$ is equivalent to the fraction $\frac{19}{45}$

$$\text{Let } x = 0.4222\dots$$

$$\Rightarrow 100x = 42.22\dots$$

$$10x = 4.22\dots$$

$$\Rightarrow 90x = 38$$

$$x = \frac{38}{90}$$

$$x = \frac{19}{45}$$

Answer: _____
(2 marks)



Q10. Prove algebraically that the recurring decimal $0.5\dot{3}2$ is equivalent to the fraction $\frac{479}{900}$

$$\text{let } x = 0.53222 \dots$$

$$\Rightarrow 10000x = 5322.22 \dots$$

$$100x = 53.22 \dots$$

$$9900x = 5269$$

$$x = \frac{5269}{9900}$$

$$x = \frac{479}{900}$$

Answer: _____
(2 marks)

Problem Questions:

Q11. Work out the following using algebra: $0.8\dot{2} + 0.\dot{3}$

$$\bullet \text{ let } x = 0.822 \dots$$

$$100x = 82.22 \dots$$

$$10x = 8.22 \dots$$

$$\Rightarrow 90x = 74$$

$$x = \frac{74}{90}$$

$$\bullet 0.8\dot{2} + 0.\dot{3} = \frac{74}{90} + \frac{1}{3}$$

$$= \frac{74 + 30}{90}$$

$$= \frac{104}{90}$$

Answer: $\frac{52}{45}$ _____
(3 marks)



Q12. Work out the following using algebra: $0.\dot{7}2 \times 0.\dot{7}$

$$\begin{aligned} \cdot \text{ let } x &= 0.7272 \dots \\ 100x &= 72.7272 \dots \\ \Rightarrow 99x &= 72 \\ x &= \frac{72}{99} \end{aligned}$$

$$\begin{aligned} \cdot \text{ let } x &= 0.77 \dots \\ 10x &= 7.7 \dots \\ \Rightarrow 9x &= 7 \\ x &= \frac{7}{9} \end{aligned}$$

$$\begin{aligned} \cdot 0.\dot{7}2 \times 0.\dot{7} &= \frac{72}{99} \times \frac{7}{9} \\ &= \frac{56}{99} \end{aligned}$$

Answer: $5\frac{6}{99}$ (3 marks)

Q13. (i) Convert the following to a mixed number using algebra: $5.\dot{3}4\dot{5}$

consider $5 + 0.345345 \dots$

$$\begin{aligned} \text{let } x &= 0.345345 \dots \\ 1000x &= 345.345 \dots \\ 999x &= 345 \\ x &= \frac{345}{999} \\ x &= \frac{115}{333} \end{aligned}$$

Answer: $5\frac{115}{333}$ (2 marks)

(ii) Hence work out $5.\dot{3}4\dot{5} - 3.\dot{2}2\dot{5}$

$3 + 0.225225 \dots$

let $x = 0.225225 \dots$

$1000x = 225.225 \dots$

$$\begin{aligned} x &= \frac{225}{1000} \\ &= \frac{9}{40} \end{aligned}$$

$$5\frac{115}{333} - 3\frac{9}{40} = 2\frac{1603}{13320}$$

Answer: $2\frac{1603}{13320}$ (2 marks)



Q14. a) A simplified fraction will be equivalent to a terminating decimal only if its denominator is just a product of 2's and/or 5's only.

(i) Use the above fact to show that $\frac{11}{28}$ will be a recurring decimal.

$$28 = 2 \times 2 \times 7$$

So this will not terminate, and be recurring

Answer: not terminate
(1 mark)

(ii) Use the above rule to decide if $\frac{583}{3500}$ is a terminating decimal.

$$3500 = 2 \times 2 \times 5 \times 5 \times 5 \times 7$$

This will not terminate.

Answer: not terminate
(1 mark)

(b) Using algebra, work out: $\frac{0.13\dot{7}4 \div 0.8}{2}$

• let $x = 0.137474 \dots$

$$100x = 13.7474 \dots$$

$$10000x = 1374.7474 \dots$$

$$9900x = 1361$$

$$x = \frac{1361}{9900}$$

$$\frac{1361}{9900} \times \frac{9}{8} = \frac{1361}{17600}$$

• let $x = 0.88 \dots$

$$10x = 8.88 \dots$$

$$9x = 8$$

$$x = \frac{8}{9}$$

Answer: $\frac{1761}{17600}$
(3 marks)



Q15. Prove that $0.\dot{0}0\dot{x}$ is equivalent to the fraction $\frac{x}{999}$ where x is a single non-zero digit.

$$\text{let } y = 0.00x00x \dots$$

$$1000y = x.00x00x \dots$$

$$999y = x$$

$$y = \frac{x}{999}$$

Answer: _____

(3 marks)

Q16. Work out the equivalent fraction to $0.\dot{y}\dot{x} - 0.\dot{x}\dot{y}$ with $y = x - 1$ where $2 \leq x \leq 9$ and $1 \leq y \leq 8$. You must justify every step.

$$\bullet \text{ let } z = 0.yxyxyx \dots$$

$$100z = yx.yxyx \dots$$

$$\Rightarrow 99z = yx$$

$$z = \frac{yx}{99}$$

$$\bullet \text{ let } w = 0.xyxxyxy$$

$$1000w = xy.xyxxyxy$$

$$\Rightarrow 999w = xy$$

$$w = \frac{xy}{999}$$

• Now consider $z - w$.

Note $yx \neq xy$, e.g. x could represent 4, y could be 3, and $34 \neq 43$.

In general, $yx - xy = 9$ [can go through the cases to show this]

$$\Rightarrow z = \frac{yx - xy}{99} \Rightarrow z = \frac{9}{99} = \frac{1}{11}$$

Answer: $\frac{1}{11}$

(4 marks)