



Completing the Square Exam Practice

Q1. a) Express $x^2 + 10x - 24$ in the form $(x + a)^2 + b$ where a and b are integers

$$(x+5)^2 - 25 - 24$$

Answer: $(x+5)^2 - 49$
(2 marks)

b) Hence solve the equation: $x^2 + 10x - 24 = 0$

$$(x+5)^2 - 49 = 0$$

$$(x+5)^2 = 49$$

$$x+5 = 7, \quad x+5 = -7$$

$$x = 2, \quad x = -12$$

Answer: 2, -12
(2 marks)

Q2. a) Express $x^2 - 8x + 15$ in the form $(x + a)^2 + b$ where a and b are integers

$$(x-4)^2 - 16 + 15$$

$$(x-4)^2 - 1$$

Answer: $(x-4)^2 - 1$
(2 marks)

b) Hence solve the equation: $x^2 - 8x + 15 = 0$

$$(x-4)^2 - 1 = 0$$

$$(x-4)^2 = 1$$

$$x-4 = 1, \quad x-4 = -1$$

$$x = 5, \quad x = 3$$

Answer: 5, 3
(2 marks)



Q3. a) Express $2x^2 - 10x - 28$ in the form $a(x + b)^2 + c$ where a , b and c are numbers to be found.

$$\begin{aligned} & 2\left[x^2 - 5x\right] - 28 \\ &= 2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] - 28 \\ &= 2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2} - 28 \\ &= 2\left(x - \frac{5}{2}\right)^2 - \frac{81}{2} \end{aligned}$$

Answer: _____
(2 marks)

b) Hence solve the equation: $2x^2 - 10x - 28 = 0$

$$\begin{aligned} 2\left(x - \frac{5}{2}\right)^2 - \frac{81}{2} &= 0 \\ 2\left(x - \frac{5}{2}\right)^2 &= \frac{81}{2} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{81}{4} \\ x - \frac{5}{2} &= \frac{9}{2}, \quad x - \frac{5}{2} = -\frac{9}{2} \end{aligned}$$

Answer: 7, -2
(2 marks)

Q4. a) Express $3x^2 + 28x + 60$ in the form $a(x + b)^2 + c$ where a , b and c are numbers to be found

$$\begin{aligned} & 3\left(x^2 + \frac{28}{3}x\right) + 60 \\ & 3\left[\left(x + \frac{28}{6}\right)^2 - \frac{784}{36}\right] + 60 \\ & 3\left(x + \frac{14}{3}\right)^2 - \frac{784}{12} + 60 \\ & 3\left(x + \frac{14}{3}\right)^2 - \frac{64}{12} \end{aligned}$$

Answer: $3\left(x + \frac{14}{3}\right)^2 - \frac{16}{3}$
(2 marks)

b) Hence solve the equation: $3x^2 + 28x + 60 = 0$

$$\begin{aligned} 3\left(x + \frac{14}{3}\right)^2 - \frac{64}{12} &= 0 \\ 3\left(x + \frac{14}{3}\right)^2 &= \frac{64}{12} \\ \left(x + \frac{14}{3}\right)^2 &= \frac{16}{9} \\ x + \frac{14}{3} &= \frac{4}{3}, \quad x + \frac{14}{3} = -\frac{4}{3} \end{aligned}$$

Answer: $-\frac{10}{3}, -6$
(2 marks)



Q5. a) Express $x^2 - 12x + 15$ in the form $(x + a)^2 + b$
where a and b are integers

$$(x-6)^2 - 36 + 15$$

$$(x-6)^2 - 21$$

Answer: $(x-6)^2 - 21$
(2 marks)

b) Hence state the co-ordinates of the minimum point of the
graph of $y = x^2 - 12x + 15 = 0$

$$(6, -21)$$

Answer: $(6, -21)$
(2 marks)



Q6. a) Express $8x - 2x^2 + 3$ in the form $a(x + b)^2 + c$ where a and b are integers

$$-2x^2 + 8x + 3$$

$$-2[x^2 - 4x] + 3$$

$$-2[(x-2)^2 - 4] + 3$$

$$-2(x-2)^2 + 8 + 3$$

$$-2(x-2)^2 + 11$$

Answer: $-2(x-2)^2 + 11$
(2 marks)

b) Hence state the co-ordinates of the maximum point of the graph of $y = 8x - 2x^2 + 3 = 0$

$$(2, 11)$$

Answer: $(2, 11)$
(2 marks)



Applied Mixed Practice Problems

Q7. A student is asked to complete the square of the expression, $2x^2 - 12x + 13$ and her answer is as follows:

$$\begin{aligned} &2x^2 - 12x + 13 \\ &= 2(x^2 - 6x) + 13 \\ &= 2(x - 3)^2 - 9 + 13 \\ &= 2(x - 3)^2 + 4 \end{aligned}$$



-9 should be $\times 2$.

Identify the mistake in her answer and work out the correct answer.

$$\begin{aligned} &2(x^2 - 6x) + 13 \\ &= 2[(x - 3)^2 - 9] + 13 \\ &= 2(x - 3)^2 - 18 + 13 \\ &= 2(x - 3)^2 - 5 \end{aligned}$$

Answer: $2(x - 3)^2 - 5$
(4 marks)



Q8. A suspension bridge consisting of 3 vertical towers connected by 2 steel connectors. The left and right connectors can be modelled by the quadratics $h = 2d^2 - 14d + 30$ and $h = d^2 - 16d + 69.5$ respectively, where h is the height (in metres) above the driveway and d is the distance from the left tower (in 10's of metres).



A service worker is positioned on the driveway underneath the lowest points on each connector. Work out the distance between the two workers.

$$2d^2 - 14d + 30$$

$$2[d^2 - 7d] + 30$$

$$2\left[\left(d - \frac{7}{2}\right)^2 - \frac{49}{4}\right] + 30$$

$$2\left(d - \frac{7}{2}\right)^2 - \frac{49}{2} + 30$$

$$2\left(d - \frac{7}{2}\right)^2 + \frac{11}{2}$$

$$\Rightarrow \text{min: } \left(\frac{7}{2}, \frac{11}{2}\right)$$

$$d^2 - 16d + 69.5$$

$$(d - 8)^2 - 64 + 69.5$$

$$(d - 8)^2 + 5.5$$

$$\Rightarrow \text{min: } (8, 5.5)$$

$$\text{Distance between workers is } 8 - \frac{7}{2} = \frac{9}{2} = \underline{4.5 \text{ m}}$$

Answer: 4.5m

(4 marks)



Q9. The height h of a ball above the ground in metres, t seconds after it is thrown, is given by the equation, $h = -4t^2 + 21t$. By completing the square, find, to 1 decimal place :

- (i) the maximum height of the ball
- (ii) the time at which the ball reaches the maximum height.

$$h = -4\left(t^2 - \frac{21}{4}t\right)$$
$$h = -4\left[\left(t - \frac{21}{8}\right)^2 - \left(\frac{21}{8}\right)^2\right]$$

$$h = -4\left(t - \frac{21}{8}\right)^2 + \frac{441}{16}$$

$$\Rightarrow \text{Max height is } \frac{441}{16}, \quad \underline{27.6 \text{ m}}$$

$$\Rightarrow \text{Reaches max height at } \frac{21}{8}, \quad \underline{2.6 \text{ seconds}}$$

Answer: 27.6 m at 2.6 sec.
(5 marks)



Q10. a) Express $2x^2 - 30x - 9$ in the form $a(x + b)^2 + c$ where a, b and c are numbers to be found.

$$2(x^2 - 15x) - 9$$

$$2\left[\left(x - \frac{15}{2}\right)^2 - \frac{225}{4}\right] - 9$$

$$2\left(x - \frac{15}{2}\right)^2 - \frac{225}{2} - 9$$

$$2\left(x - \frac{15}{2}\right)^2 - \frac{243}{2}$$

Answer: $2\left(x - \frac{15}{2}\right)^2 - \frac{243}{2}$
(2 marks)

b) Hence solve the equation: $2x^2 - 30x - 9 = 0$ giving your answer in surd form.

$$2\left(x - \frac{15}{2}\right)^2 - \frac{243}{2} = 0$$

$$2\left(x - \frac{15}{2}\right)^2 = \frac{243}{2}$$

$$\left(x - \frac{15}{2}\right)^2 = \frac{243}{4}$$

$$x - \frac{15}{2} = \frac{\sqrt{243}}{2}, \quad x - \frac{15}{2} = -\frac{\sqrt{243}}{2}$$

$$x - \frac{15}{2} = \frac{9\sqrt{3}}{2}, \quad x - \frac{15}{2} = -\frac{9\sqrt{3}}{2}$$

Answer: $\frac{9\sqrt{3} + 15}{2}, \frac{-9\sqrt{3} + 15}{2}$
(2 marks)