



Algebraic Fractions Exam Practice

Simplifying Fractions

Q1. Simplify fully $\frac{x+5}{x^2+3x-10}$

$$\frac{x+5}{(x+5)(x-2)} = \frac{\cancel{(x+5)}}{\cancel{(x+5)} \times (x-2)} = \frac{1}{x-2}$$

Answer: $\frac{1}{x-2}$
(2 marks)

Q2. Simplify fully $\frac{x^2-4x}{x^2+4x-32}$

$$\frac{x(x-4)}{(x+8)(x-4)} = \frac{\cancel{(x-4)}}{\cancel{(x-4)} (x+8)} = \frac{x}{x+8}$$

Answer: $\frac{x}{x+8}$
(2 marks)



Q3. Simplify fully $\frac{x^2 - 9}{x^2 - 10x + 21}$

$$\frac{(x+3)(x-3)}{(x-3)(x-7)} = \frac{\cancel{x-3}}{\cancel{x-3}} \times \frac{x+3}{x-7}$$

Answer: $\frac{x+3}{x-7}$
(2 marks)

Q4. Simplify fully $\frac{x^2 - 4x}{x^2 + 4x - 32}$

$$\frac{x(x-4)}{(x+8)(x-4)} = \frac{\cancel{x-4}}{\cancel{x-4}} \times \frac{x}{x+8}$$

Answer: $\frac{x}{x+8}$
(2 marks)



Q5. Express $\frac{4x^2 - 9}{2x^2 + 13x + 15} \div \frac{2x^2 - 3x}{x^2}$ in the form $\frac{x}{x+a}$ where a is to be found.

$$\frac{(2x-3)(2x+3)}{(2x+3)(x+5)} \times \frac{x^2}{2x^2-3x}$$
$$= \frac{(2x-3)(2x+3)}{(2x+3)(x+5)} \times \frac{x^2}{x(2x-3)} = \frac{\cancel{2x-3} \times \cancel{2x+3} \times \cancel{x} \times \cancel{x}}{\cancel{2x+3} \times \cancel{x} \times \cancel{2x-3}} = \frac{x}{x+5}$$

Answer: $\frac{x}{x+5}$
(3 marks)

Q6. Simplify fully $\frac{1}{x} + \frac{3}{x+1}$

$$\frac{1(x+1) + 3x}{x(x+1)} = \frac{x+1+3x}{x(x+1)} = \frac{4x+1}{x(x+1)}$$

Answer: $\frac{4x+1}{x(x+1)}$
(3 marks)



Q7. Simplify fully: $\frac{10y^2 + 15y}{5y^2} \times \frac{y^2 + 4y}{2y + 3}$

$$\frac{5y(2y+3)}{5y^2} \times \frac{y(y+4)}{2y+3}$$

$$\frac{\cancel{5y}}{\cancel{5y}} \times \frac{\cancel{2y+3}}{\cancel{2y+3}} \times \frac{\cancel{y}}{\cancel{y}} \times (y+4)$$

Answer: y + 4
(3 marks)

Q8. Write the sum $1 - \frac{1}{x-2} + \frac{3}{x^2-x-2}$ in the form $\frac{x+c}{x+d}$ where c and d are numbers to be found.

$$1 - \frac{1}{x-2} + \frac{3}{(x-2)(x+1)} = \frac{1(x-2)(x+1) - 1(x+1) + 3}{(x-2)(x+1)}$$

$$= \frac{x^2 - x - 2 - x - 1 + 3}{(x-2)(x+1)} = \frac{x^2 - 2x}{(x-2)(x+1)}$$

$$= \frac{x(x-2)}{(x-2)(x+1)} = \frac{\cancel{x-2}}{\cancel{x-2}} \times \frac{x}{x+1}$$

Answer: $\frac{x}{x+1}$
(4 marks)



Solving Fractional Equations

Q9. Solve the equation: $\frac{x-18}{x^2+6x-3} = 2$

$$x - 18 = 2(x^2 + 6x - 3)$$

$$x - 18 = 2x^2 + 12x - 6$$

$$2x^2 + 11x + 12 = 0$$

$$(2x + 3)(x + 4) = 0$$

$$x = -\frac{3}{2} \quad x = -4$$

Answer: $x = -\frac{3}{2} \quad x = -4$
(3 marks)

Q10. Solve the equation: $\frac{1}{x^2+3x-10} + \frac{x+5}{x^2+3x-10} = 4$, giving your answers to 3 s.f.

$$\frac{x+6}{x^2+3x-10} = 4 \Rightarrow x+6 = 4(x^2+3x-10)$$

$$\Rightarrow x+6 = 4x^2+12x-40 \Rightarrow 4x^2+11x-46 = 0$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(4)(-46)}}{2(4)} = 2.28 \text{ or } -5.03$$

Answer: $2.28 \text{ or } -5.03$
(4 marks)



Q11. a) Write $\frac{y}{y-4} - \frac{28}{y^2-y-12}$ as one fraction, fully simplifying your answer.

$$\frac{y}{y-4} - \frac{28}{(y-4)(y+3)} = \frac{y(y+3) - 28}{(y-4)(y+3)}$$

$$= \frac{y^2 + 3y - 28}{(y-4)(y+3)} = \frac{(y+7)(y-4)}{(y+3)(y-4)}$$

Answer: $\frac{y+7}{y+3}$ (3 marks)

b) Hence solve the equation $\frac{y}{y-4} - \frac{28}{y^2-y-12} = 5$

$$\frac{y+7}{y+3} = 5 \Rightarrow y+7 = 5y+15$$

$$\Rightarrow 4y = -8 \Rightarrow y = -2$$

Answer: $y = -2$ (3 marks)

Q12. Solve the equation $\frac{14}{a+1} - \frac{8}{3a-2} = 2$

$$\frac{14(3a-2) - 8(a+1)}{(a+1)(3a-2)} = 2 \Rightarrow \frac{42a - 28 - 8a - 8}{(a+1)(3a-2)} = 2$$

$$\Rightarrow 34a - 36 = 2(a+1)(3a-2)$$

$$\Rightarrow 34a - 36 = 2(3a^2 - 2a + 3a - 2)$$

$$\Rightarrow 34a - 36 = 6a^2 + 2a - 4$$

$$\Rightarrow 6a^2 - 32a + 32 = 0$$

$$\Rightarrow 3a^2 - 16a + 16 = 0$$

$$\Rightarrow (3a-4)(a-4) = 0$$

Answer: $a = \frac{4}{3}$ or $a = 4$ (4 marks)



Q13. a) Solve $\frac{3}{x-4} - \frac{5}{x} = 2$

$$\frac{3x - 5(x-4)}{x(x-4)} = 2 \Rightarrow \frac{3x - 5x + 20}{x(x-4)} = 2$$

$$\Rightarrow -2x + 20 = 2x(x-4) \Rightarrow -2x + 20 = 2x^2 - 8x$$

$$\Rightarrow 2x^2 + 6x - 20 = 0 \Rightarrow x^2 + 3x - 10 = 0$$

$$\Rightarrow (x+5)(x-2) = 0$$

Answer: $x = -5$ or 2
(3 marks)

b) Hence solve the equation $\frac{3}{2x+1} - \frac{5}{2x+5} = 2$

$$\frac{3(2x+5) - 5(2x+1)}{(2x+1)(2x+5)} = 2 \Rightarrow \frac{6x + 15 - 10x - 5}{(2x+1)(2x+5)} = 2$$

$$\Rightarrow -4x + 10 = 2(4x^2 + 10x + 2x + 5)$$

$$\Rightarrow -4x + 10 = 8x^2 + 24x + 10$$

$$\Rightarrow 8x^2 + 28x = 0$$

$$\Rightarrow 4x(2x+7) = 0$$

Answer: $x = 0, -\frac{7}{2}$
(2 marks)

Q14. Given that the ratio $8 : (x + 4)$ is equivalent to the ratio $28 : (x - 3)$, find the value of x .

$$\frac{8}{x+4} = \frac{28}{x-3} \Rightarrow 8(x-3) = 28(x+4)$$

$$\Rightarrow 8x - 24 = 28x + 112 \Rightarrow 20x = -136$$

$$\Rightarrow x = \frac{-136}{20} = -6.8$$

Answer: $x = -6.8$
(4 marks)



Q15. a) Solve $\frac{2}{x} - \frac{36}{x(x+8)} = 2$

$$\frac{2(x+8) - 36}{x(x+8)} = 2 \Rightarrow 2x + 16 - 36 = 2x(x+8)$$

$$2x - 20 = 2x^2 + 16x \Rightarrow 2x^2 + 14x + 20 = 0$$
$$\Rightarrow x^2 + 7x + 10 = 0 \Rightarrow (x+5)(x+2) = 0$$

Answer: $x = -5$ or -2
(4 marks)

b) Hence solve the equation $\frac{2}{3x+1} - \frac{36}{(3x+1)(3x+9)} = 2$

$$3x + 1 = -5 \Rightarrow 3x = -6 \Rightarrow x = -2$$

$$3x + 1 = -2 \Rightarrow 3x = -3 \Rightarrow x = -1$$

Answer: $x = -2$ or -1
(2 marks)

Q16. Solve $2 + \frac{x+11}{2x^2-5x-3} - \frac{x-1}{x-3} = 0$, where $x \neq -3$, $x \neq -\frac{1}{2}$

$$2 + \frac{x+11}{(2x+1)(x-3)} - \frac{x-1}{x-3} = 0 \Rightarrow \frac{2(2x+1)(x-3) + (x+11) - (x-1)(2x+1)}{(2x+1)(x-3)}$$

$$\Rightarrow \frac{2(2x^2 - 5x - 3) + x + 11 - (2x^2 - x - 1)}{(2x+1)(x-3)} = 0$$

$$\Rightarrow \frac{4x^2 - 10x - 6 + x + 11 - 2x^2 + x + 1}{(2x+1)(x-3)} = 0$$

$$\Rightarrow 2x^2 - 8x + 6 = 0 \Rightarrow x^2 - 4x + 3 = 0$$

$$\Rightarrow (x-3)(x-1) = 0$$

Answer: $x = 1$ or 3
(5 marks)