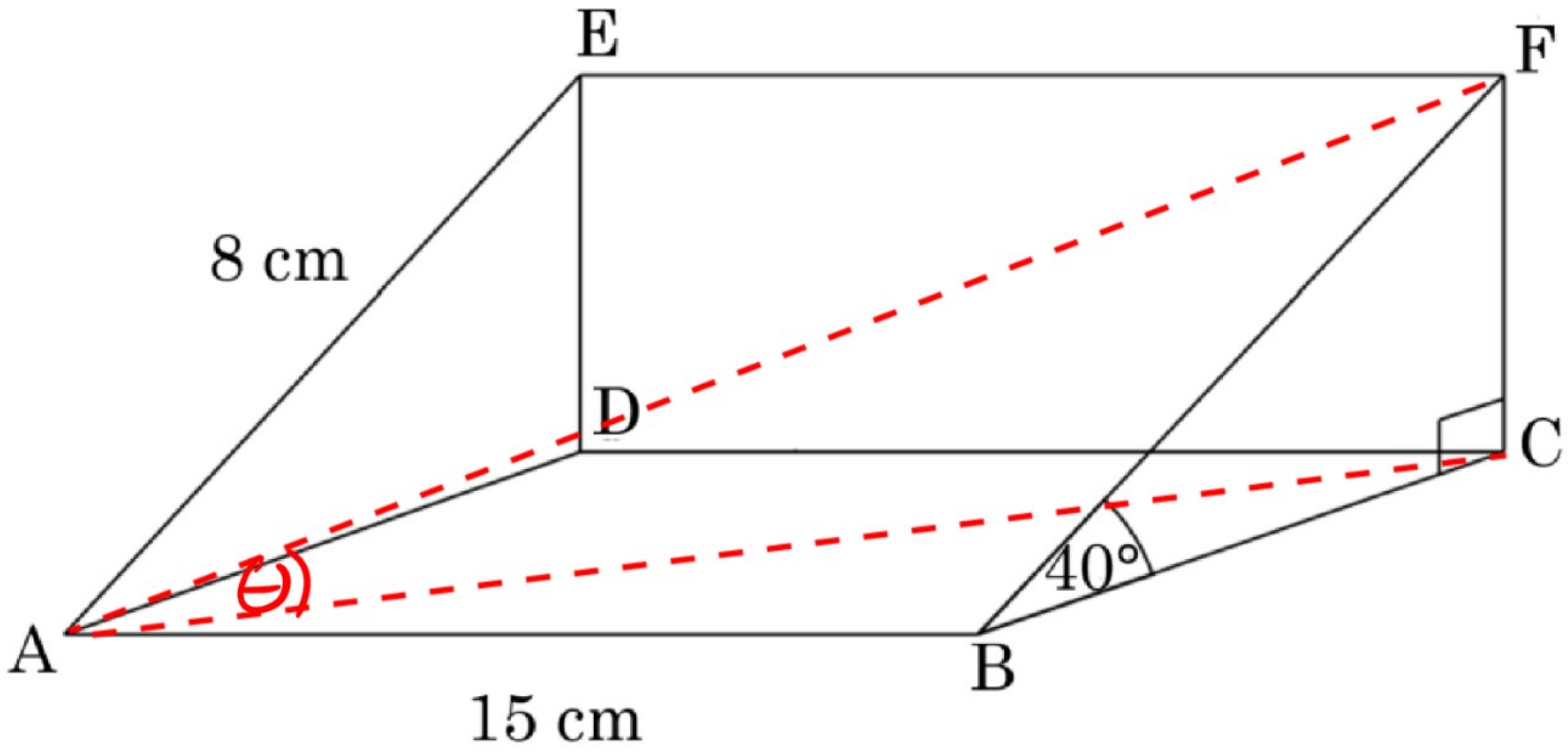
# 3d Trigonometry Exam Practice

Q1. Find the angle between AF and the plan ABCD.




$\sin(40) = \frac{FC}{8}$

$\Rightarrow FC = 8\sin(40)$

$\cos(40) = \frac{BC}{8}$

$\Rightarrow BC = 8\cos(40)$

• $(AC)^2 = 15^2 + (8\cos(40))^2$ by pythagoras

$\Rightarrow AC = 16.203\ldots$

• $\tan\theta = \frac{FC}{AC}$ $\Rightarrow \tan\theta = 0.317$

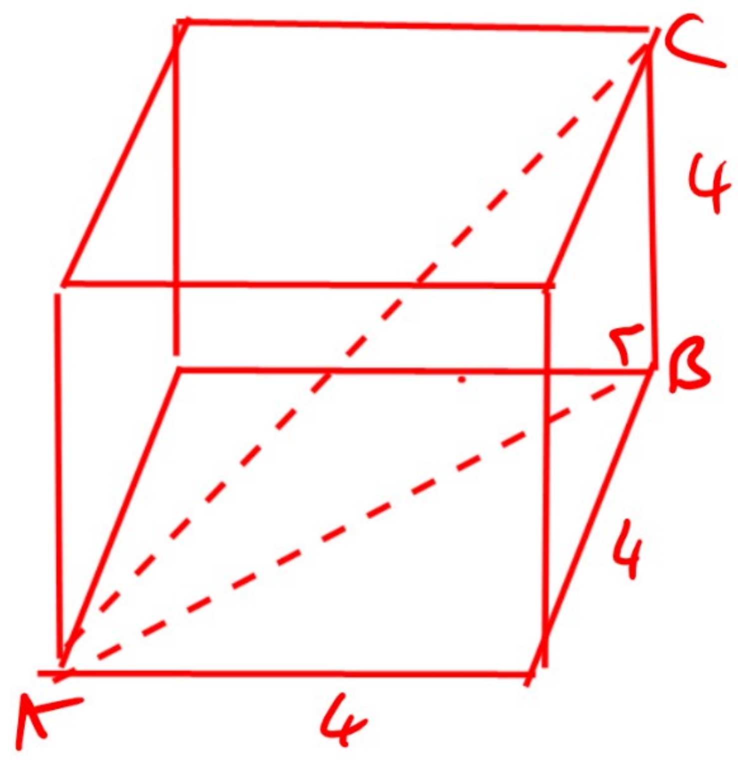
$\theta = 17.607\ldots$

Answer: 17.6°

(2 marks)

Q2. A cube has side length 4 cm. Work out the longest direct distance between any two vertices, giving your answer in exact form.

(find AC)




$\cdot\ (AB)^2 = 4^2 + 4^2$ (pythagoras)

$AB = \sqrt{32}$

$(AC)^2 = (AB)^2 + 4^2$ (pythagoras)

$= 32 + 4^2$

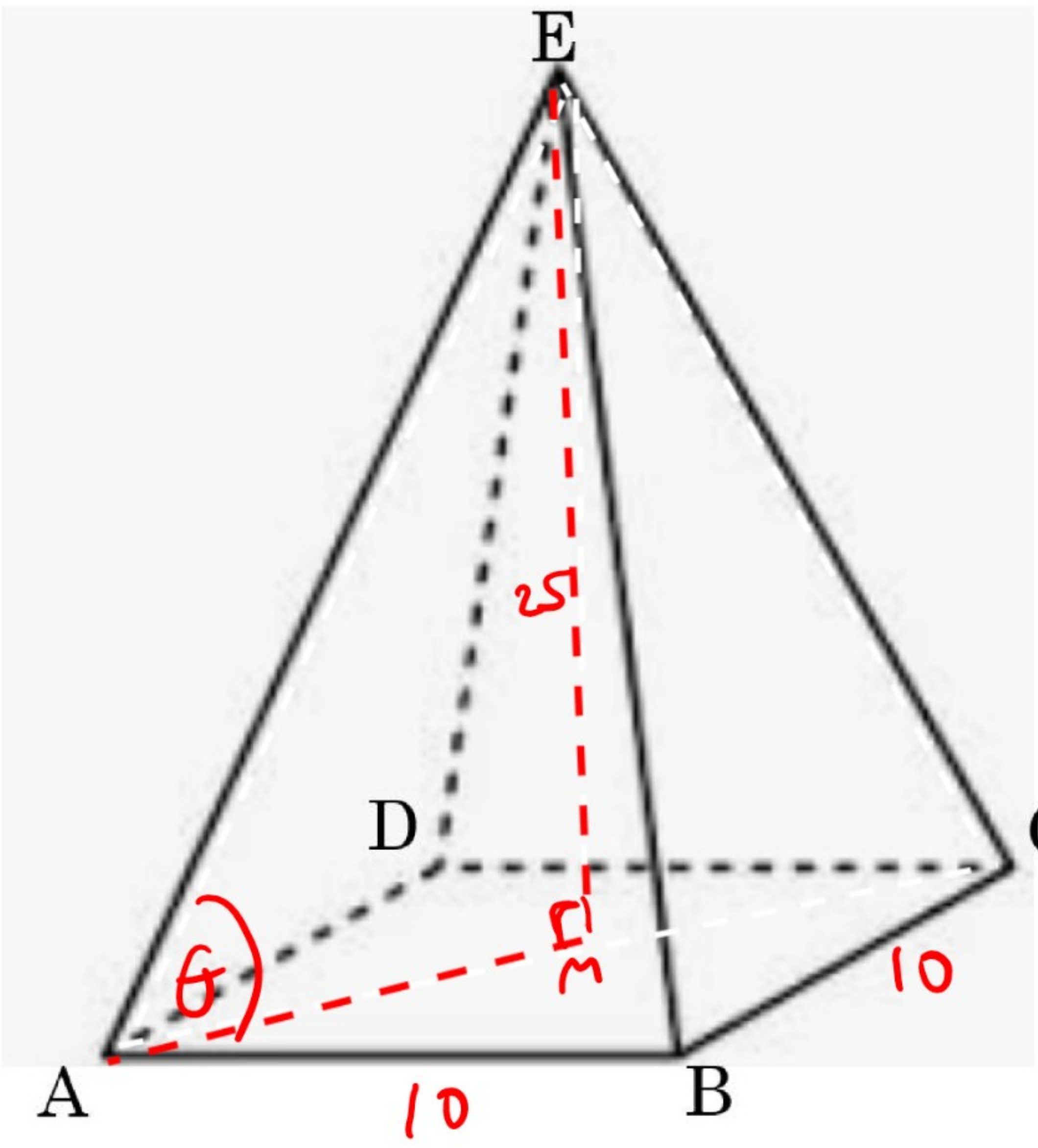
$= 48$, so $AC = \sqrt{48}$

$= 4\sqrt{3}$

Answer: $4\sqrt{3}$

(4 marks)

Q3. ABCDE is a square based pyramid. AB = 10 cm, & E is 25 cm vertically above the base ABCD. Find the size of angle EAC to 1 decimal place.

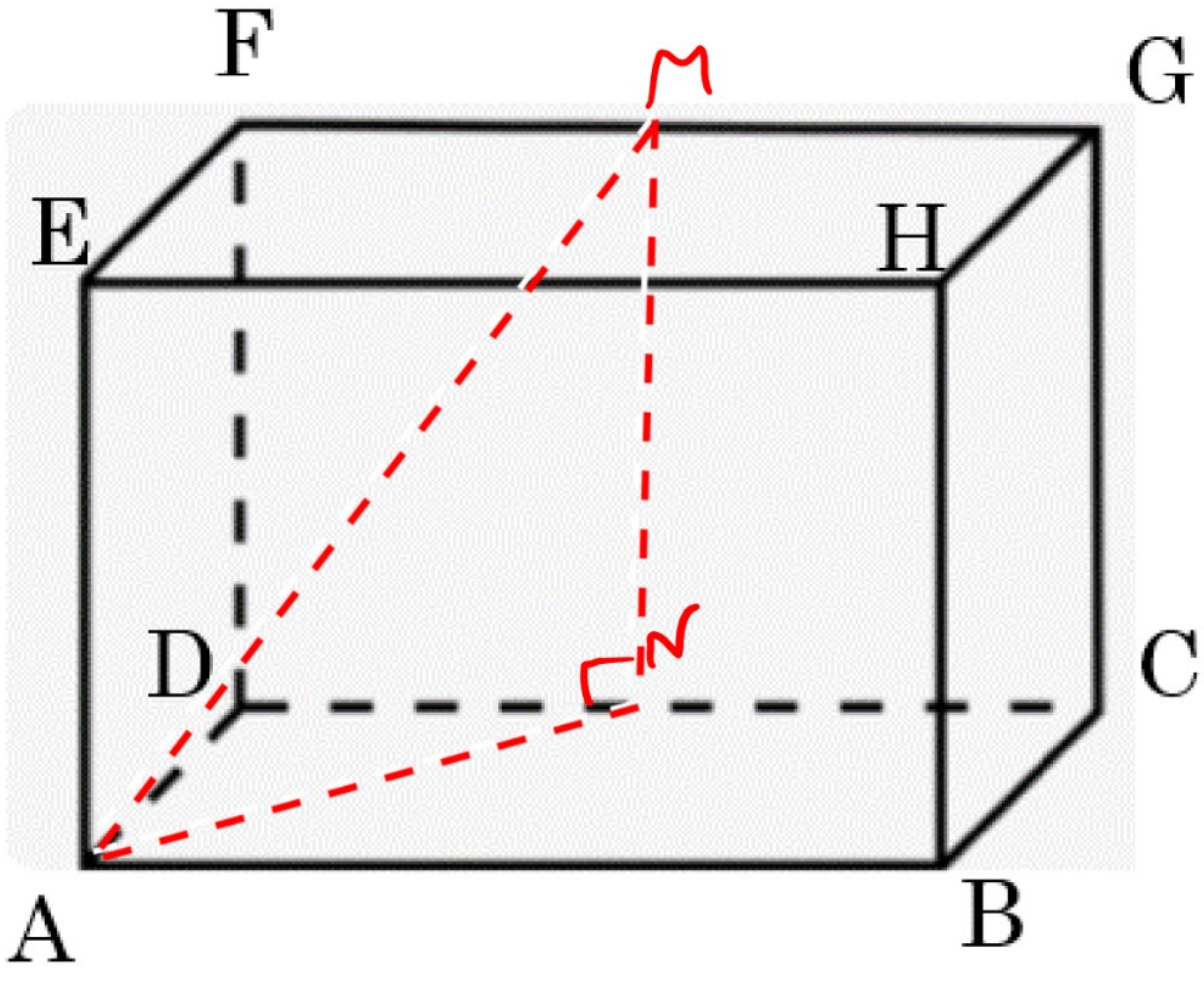



- $(AC)^2 = 10^2 + 10^2$
- $AC = \sqrt{200}$
  $= 10\sqrt{2}$
- $AM = \frac{10\sqrt{2}}{2}$
  $= 5\sqrt{2}$
- $\theta = \tan^{-1}\left(\frac{25}{5\sqrt{2}}\right)$
  $= 74.20\ldots$

Answer: $74.2^\circ$

(4 marks)

Q4. In this cuboid, AB = 18, BC = BH = 12, and M is the mid-point of FG.




Find the length of AM to 3 s.f.

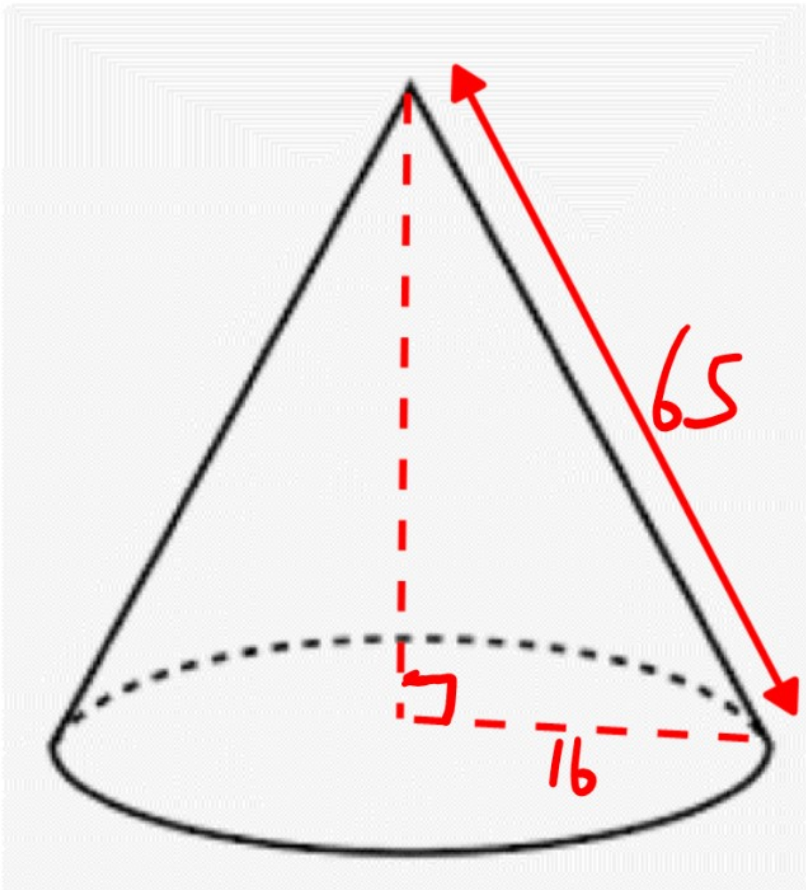
- $DN = \frac{1}{2}(AB) \Rightarrow DN = 9$
- $(AN)^2 = (AD)^2 + (DN)^2$
  $= 12^2 + 9^2$
  $AN = \sqrt{225}$
  $= 15$
- $(AM)^2 = (AN)^2 + (MN)^2$
  $= 15^2 + 12^2$
  $AM = \sqrt{369}$

$AM = 19.209\ldots$

Answer: 19.2

(4 marks)

Q5. In the cone below, the circular base has diameter 32 cm and the slanting height is 65 cm. Find the volume to 1 d.p.




- $h^2 = 65^2 - 16^2$

  $h = \sqrt{3969}$

  $h = 63$

- $V = \frac{1}{3}\pi r^2 h$

  $= \frac{1}{3}\pi (16)^2 63$

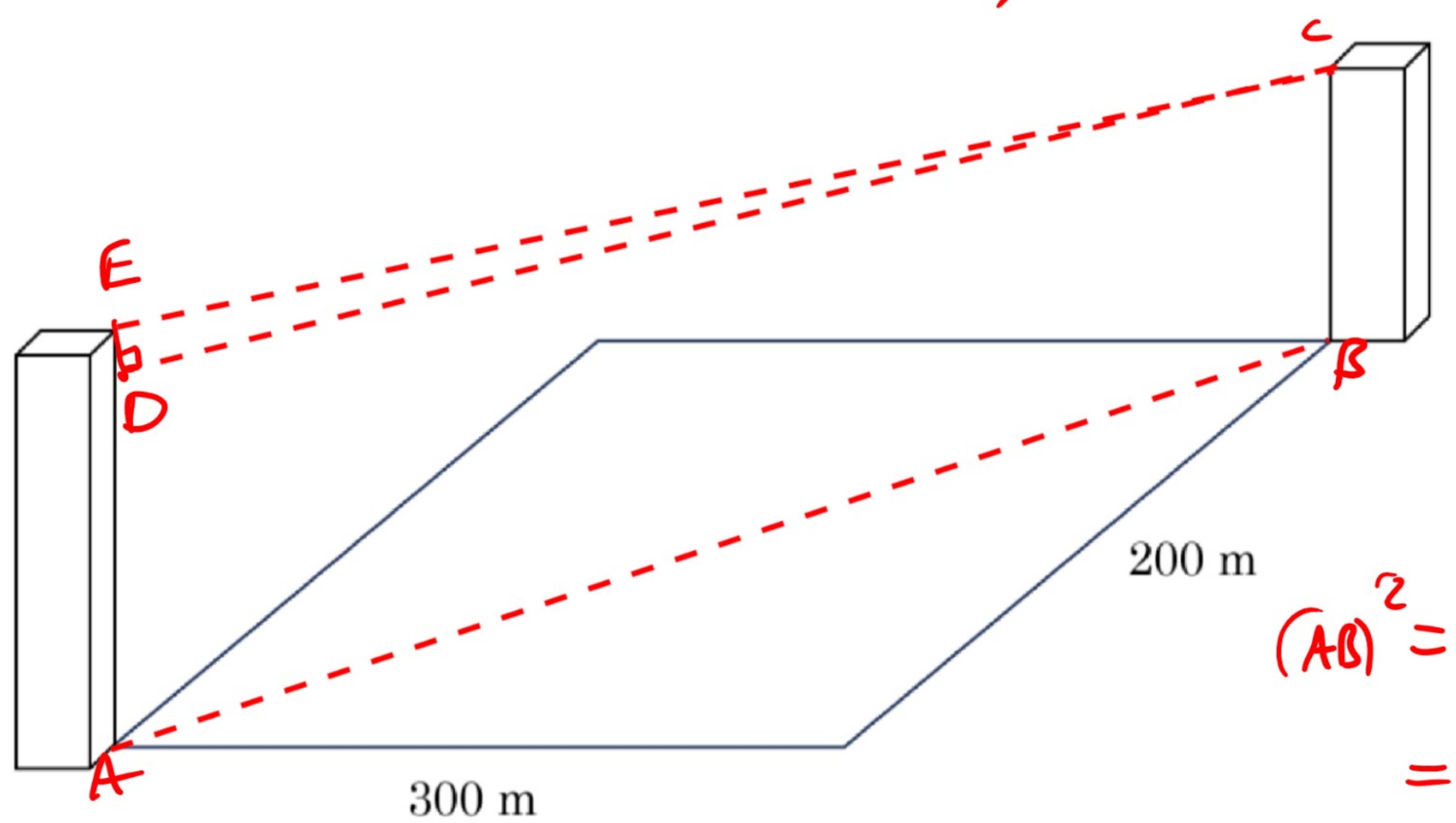
  $= 16889.20..$

Answer: $16889.2 \text{ cm}^3$

(3 marks)

Q6. A stunt-man is going to connect the nearest corner of each tower with a wire, and slide between. The towers are 55 m and 45 m tall.

Find the distance he will travel. (CE)




- $DE = 55 - 45$

  $= 10$

$(AB)^2 = \sqrt{300^2 + 200^2}$

$= \sqrt{130000}$

$= CD$

$(CE)^2 = (CD)^2 + (DE)^2$

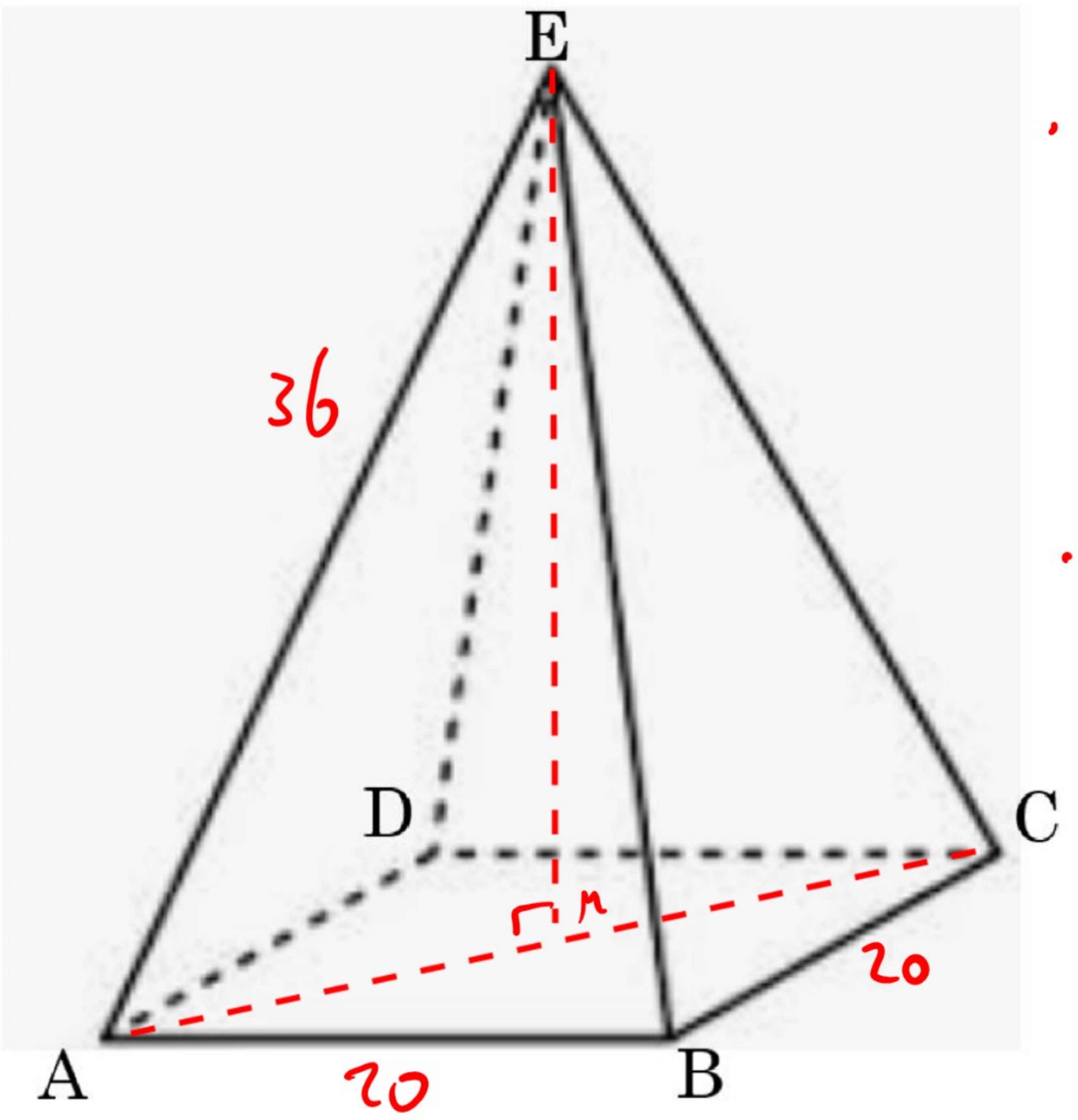
$= 130000 + 100$

$= 130100 \rightarrow CE \sqrt{130100}$

Answer: 360.69 m

(5 marks)

Q7. ABCDE is a square based pyramid where AE = 36 cm, EAC is 55°, and AB = 20 cm. Find the volume of the pyramid to 3 s.f.




$\cdot\ (AC)^2 = \sqrt{20^2 + 20^2}$

$= \sqrt{800}$

$= 20\sqrt{2}$

$\cdot\ AM = \frac{1}{2}(AC)$

$= 10\sqrt{2}$

$\cdot\ E\hat{A}C = \cos^{-1}\left(\frac{10\sqrt{2}}{36}\right)$

$= 38.216...$

$V = \frac{1}{3} \times \text{base area} \times \text{perpendicular height}$

$= \frac{1}{3} \times 20^2 \times 38.216...$

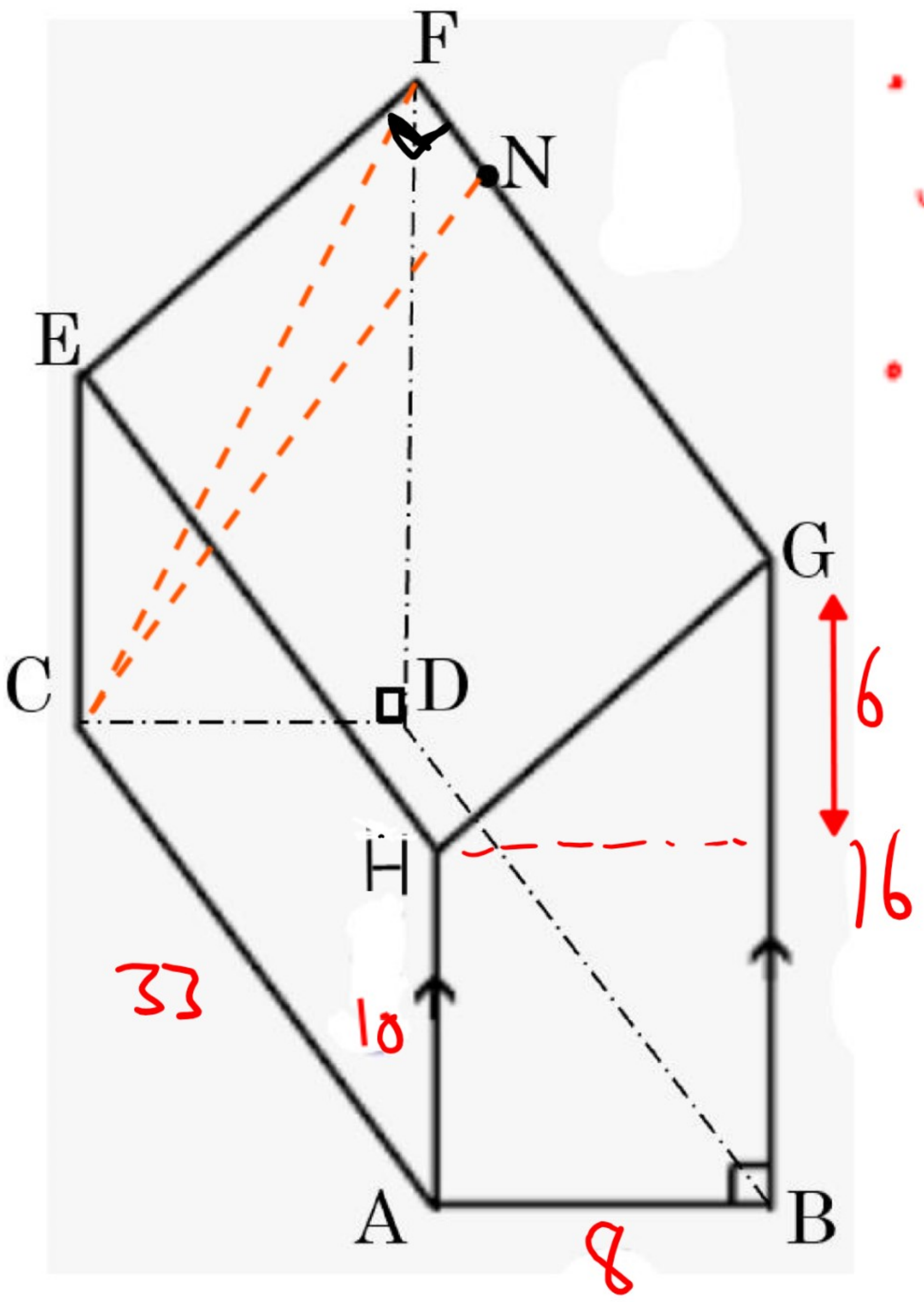
$= 5095.589...$

Answer: 5100 (3 s.f.)

(4 marks)

Q8. Below is a prism where: AC = 33,
CD = 8,
GN : NF = 8 : 3,
AH = 10,
DF = 16

Find angle CAN correct to 2 d.p.




• we find CN, AN and use the cosine rule.

• GN $= \frac{8}{11}(33) = 24$ NF $= 33-24 = 9$

• HG $= \sqrt{8^2+6^2} = 10$

AG $= \sqrt{8^2+16^2} = 8\sqrt{5}$

• In $\triangle CFN$, $FN = 9$
$CF = 8\sqrt{5}$ $(= AG)$

$\Rightarrow (CN)^2 = (CF)^2 + (FN)^2 \Rightarrow CN = \sqrt{401}$

• Similarly, $AN = 8\sqrt{14}$

• $a^2 = b^2 + c^2 - 2bc\cos A$

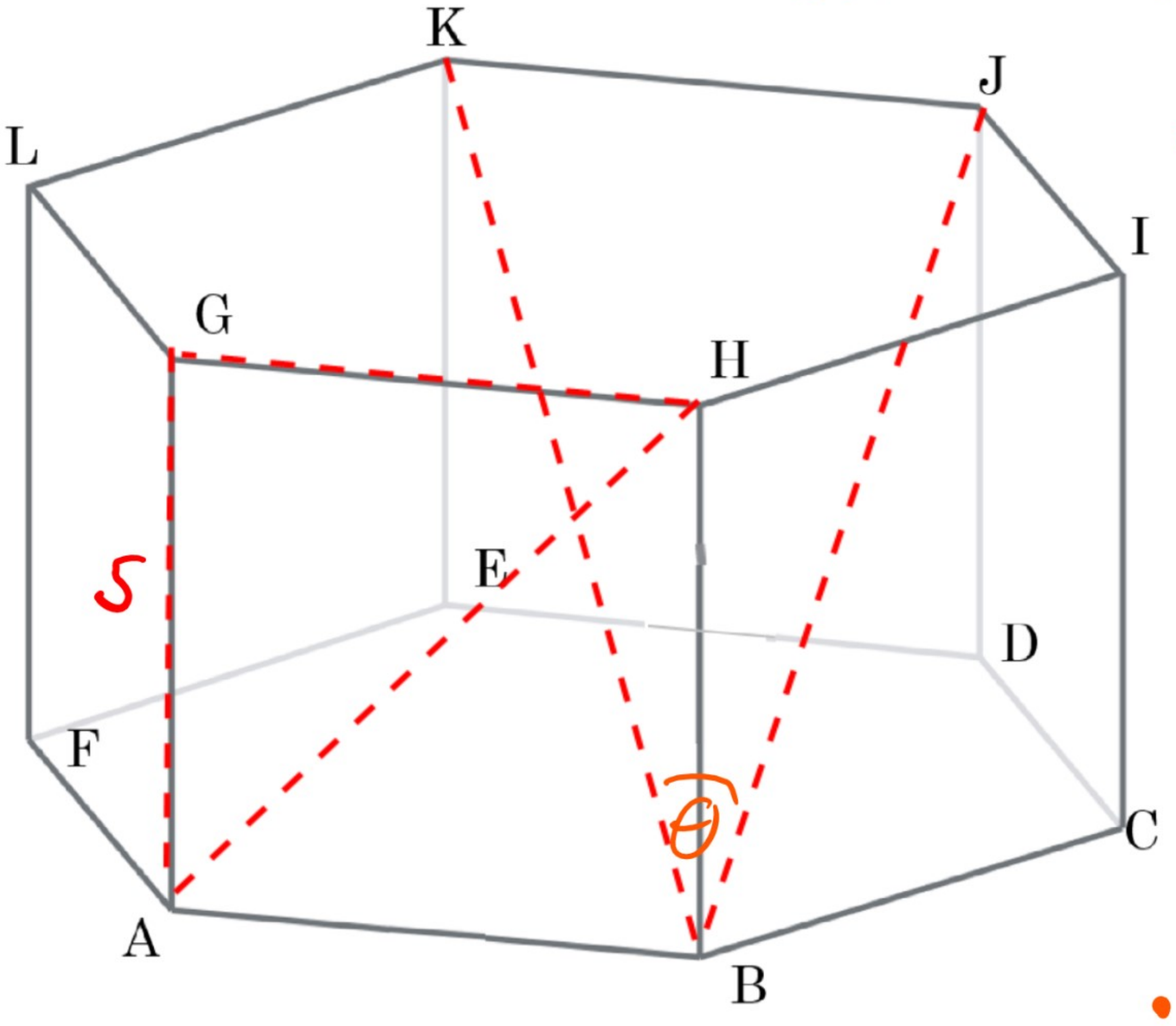
where $a = 33$, $b = \sqrt{401}$, $c = 8\sqrt{14}$ $\Rightarrow CNA = \cos^{-1}(0.173..)$

Answer: 80.01°

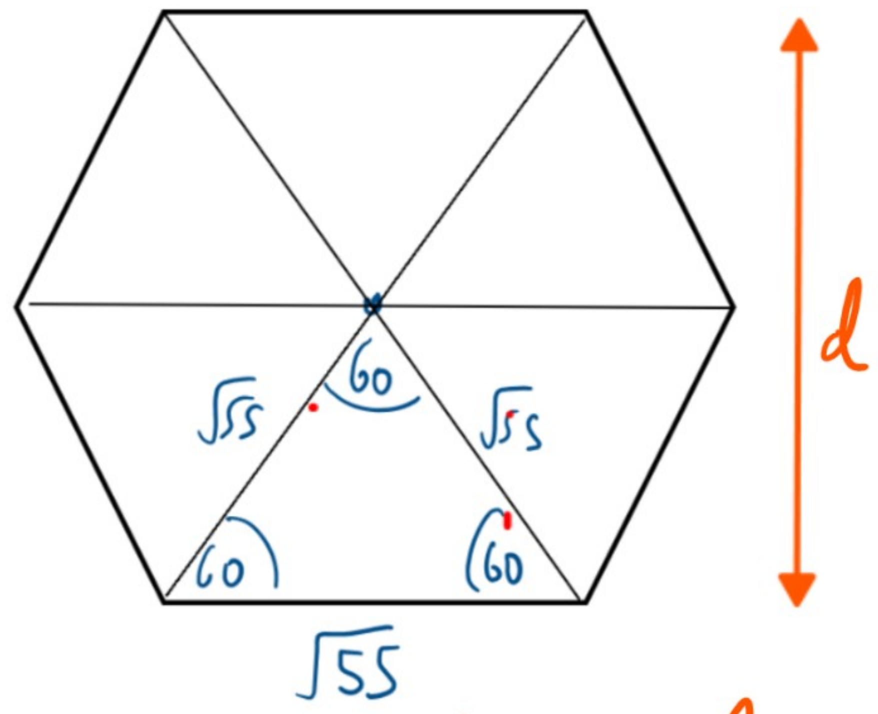
(7 marks)

Q9. In the regular hexagonal prism below, AH = $\sqrt{80}$, AG = 5.
Find angle KBJ to 1 d.p.

(We find BK, BJ, KJ and use the cosine rule)




• $GH = \sqrt{(\sqrt{80})^2 - 5^2}$
$= \sqrt{55}$ $(= KJ)$




• In $\Delta KEB$, $EB = 2\sqrt{55}$ $KE = 5$
$\Rightarrow (KB)^2 = (2\sqrt{55})^2 + 5^2 \Rightarrow KB = 7\sqrt{5}$

• $d = 2 \times \sin(60)\sqrt{55}$ $(= \sqrt{165})$

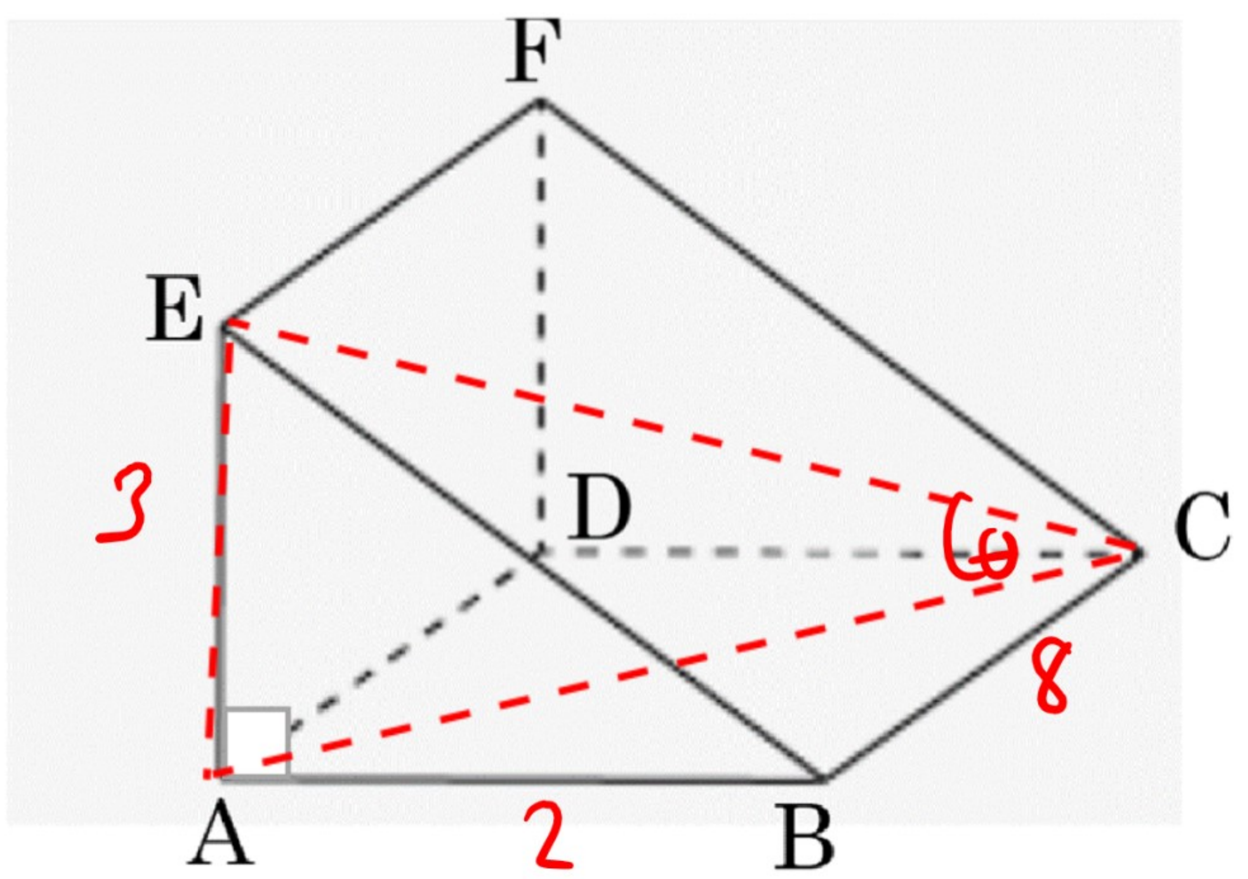
• In BJD, $BJ = \sqrt{(2\sin(60)\sqrt{55})^2 + 5^2} = \sqrt{190}$

• $a^2 = b^2 + c^2 - 2bc\cos A$
with $a = \sqrt{55}$, $b = 7\sqrt{5}$, $c = \sqrt{190}$
$\Rightarrow \theta = \cos^{-1}(0.880\ldots)$

Answer: 28.3°

(6 marks)

Q10. In the prism, AB : EA : BC is 2 : 3 : 8. Find angle ACE to 1 d.p.




We can let $AB = 2$,
$EA = 3$,
$BC = 8$

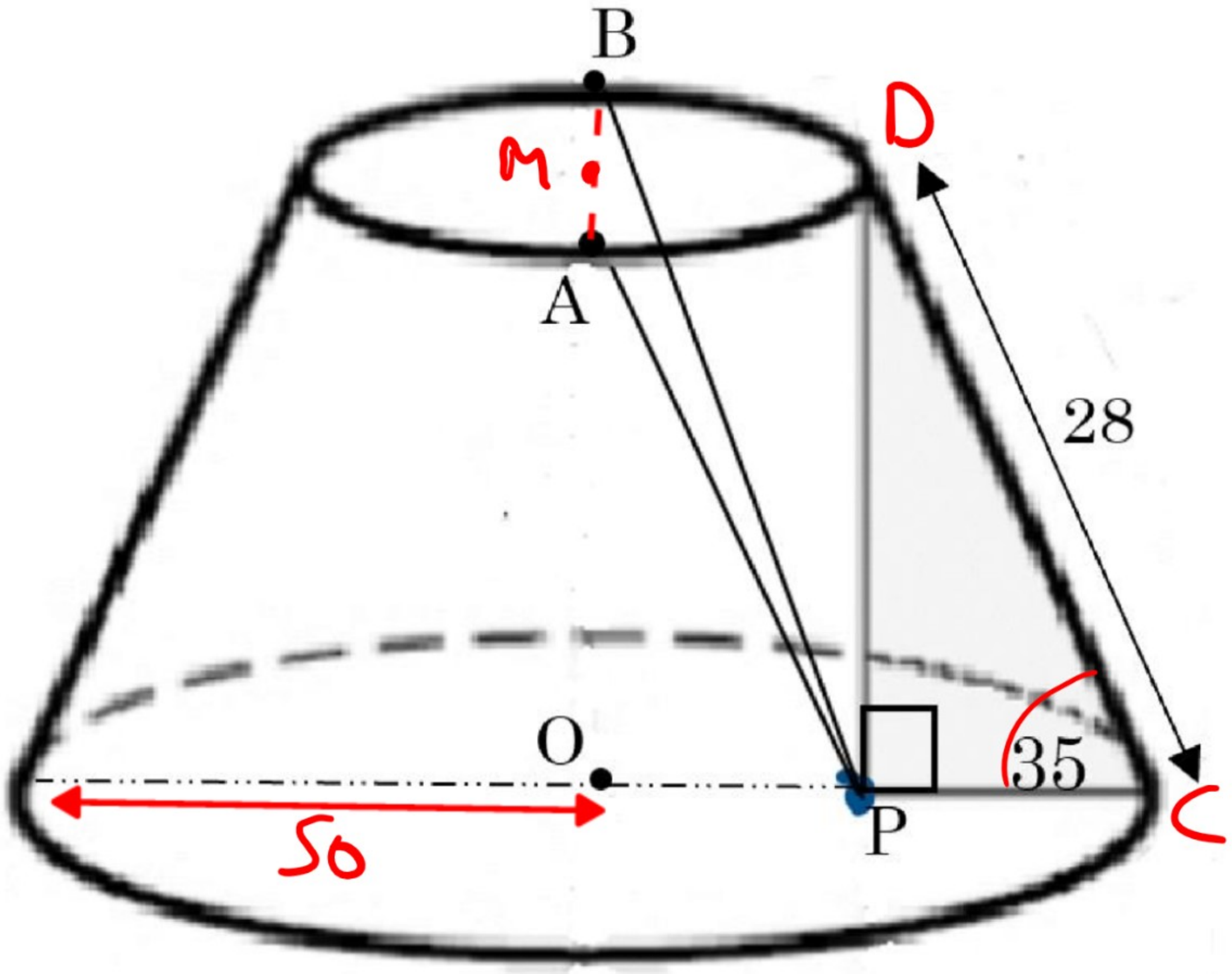
$AC = \sqrt{2^2 + 8^2}$
$= \sqrt{68}$

$\theta = \tan^{-1}\left(\frac{3}{\sqrt{68}}\right)$
$= 19.99\ldots$

Answer: 20.0

(4 marks)

Q11. In the frustum, AB is a diameter of the top, O is the centre of the base, which has diameter 100. Find angle PAB to 4 s.f.

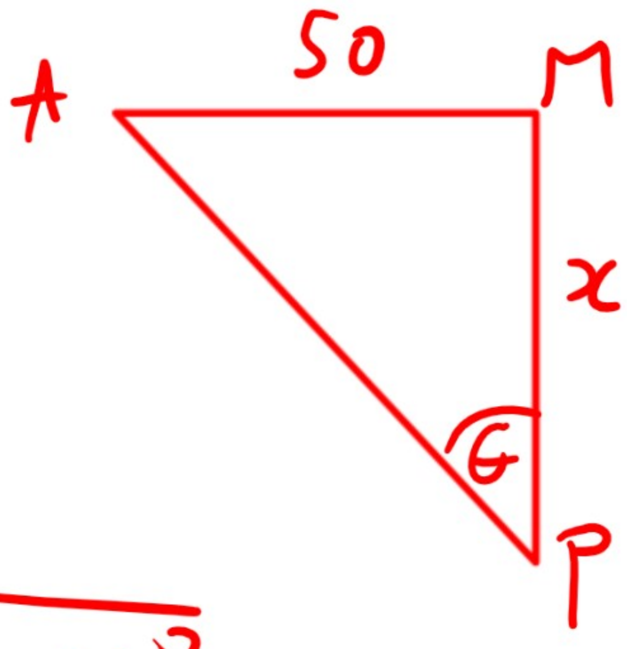



• PC = 28cos(35) , OM = 28sin(35)

• OP = 50 − 28cos(35).

• By symmetry PAB is isoceles. (PA = PB)

∴ PAB = 2 × APM




• For x, use ΔOMP :

$x = \sqrt{(28\sin 35)^2 + (50 - 28\cos 35)^2}$

$= 31.470\ldots$

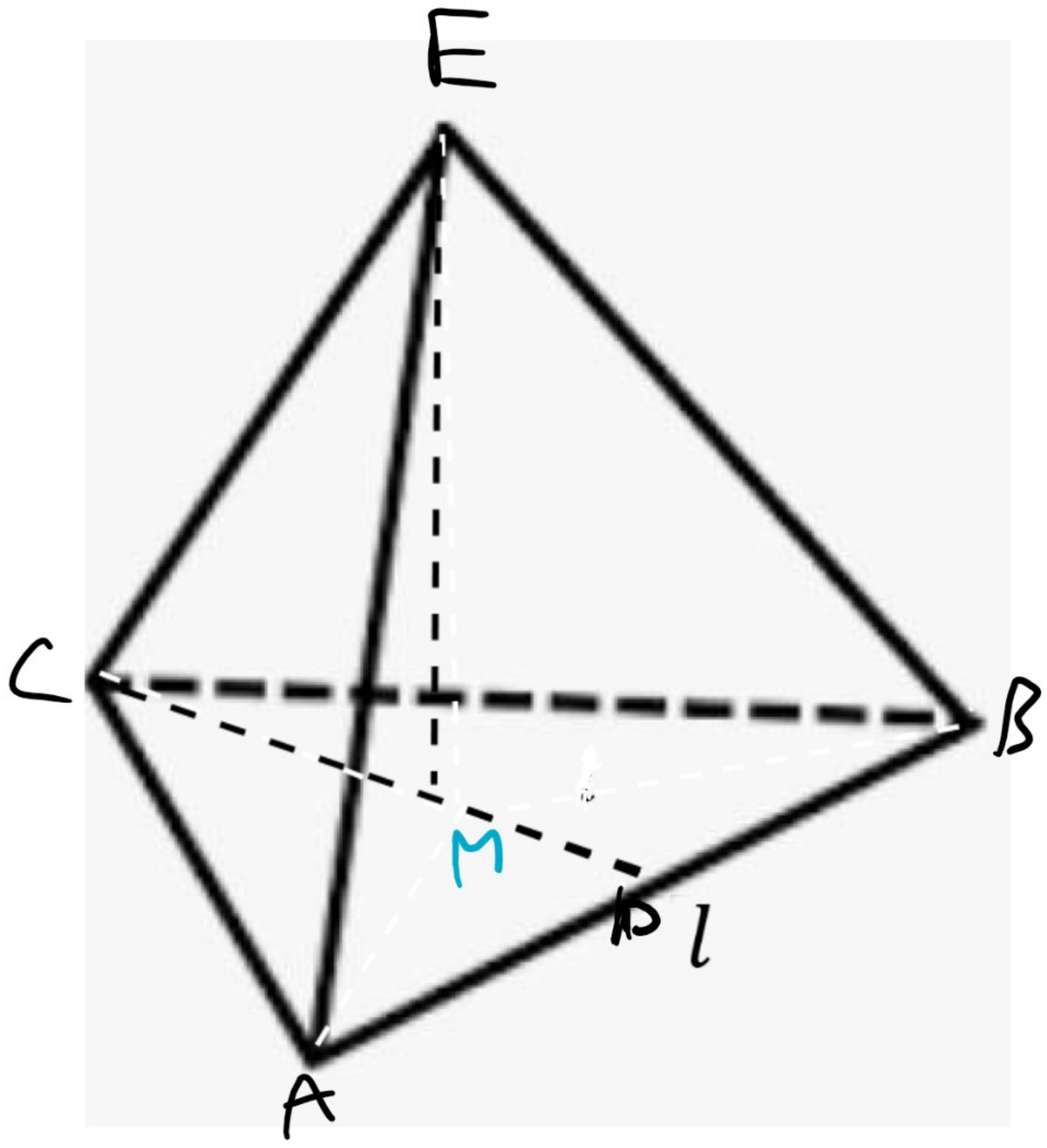
• $\theta = \tan^{-1}\left(\frac{50}{31.470\ldots}\right)$

$= 57.8 \Rightarrow PAB = 2 \times 57.8 = 115.62\ldots$

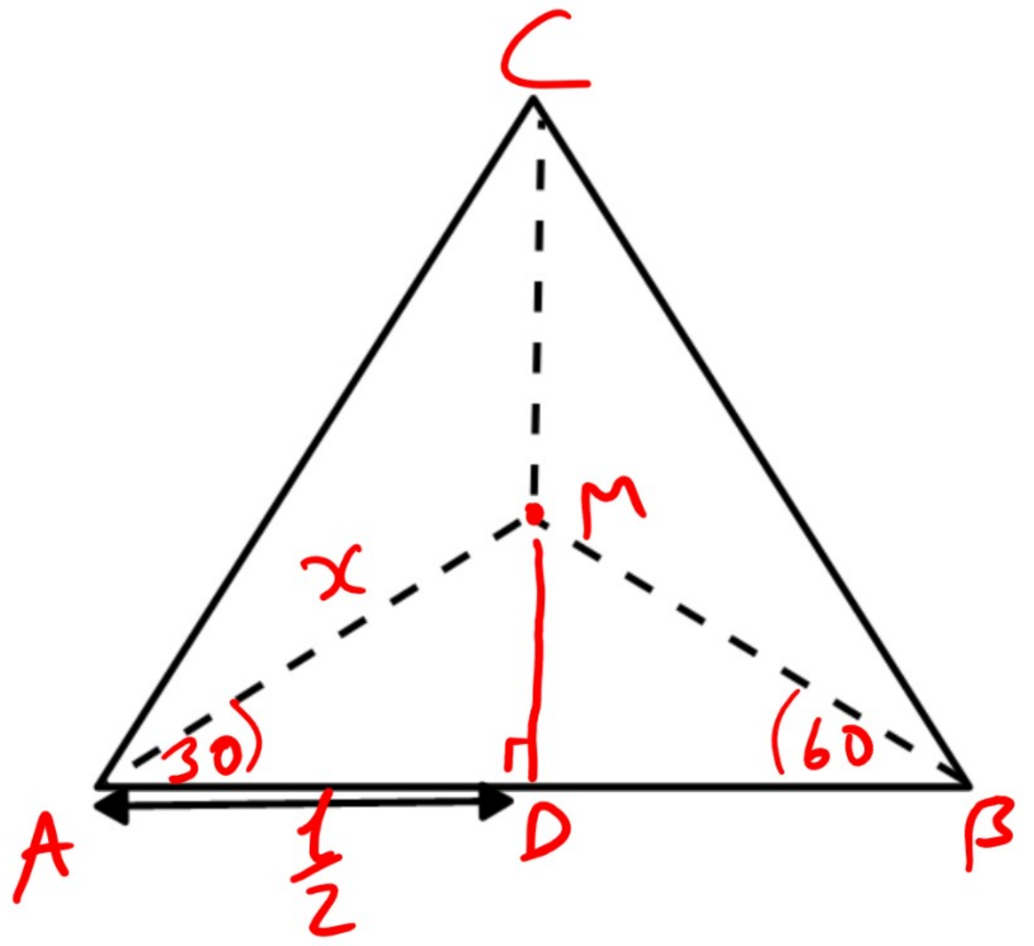
Answer: 115.6°

(6 marks)

Q12. The tetrahedron below has 4 identical faces which are equilateral triangles. Find the vertical height of the tetrahedron in terms of $l$ giving your answer in the form $\frac{\sqrt{k}}{3}$ for some $k$.




Each angle at M is $\frac{360}{3} = 120°$




$\cos 30 = \frac{l}{2} \div x$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{l}{2x}$

$\Rightarrow x = \frac{l}{\sqrt{3}}$

In AME, $(ME)^2 = (AE)^2 - (AM)^2$

$= l^2 - \left(\frac{l}{\sqrt{3}}\right)^2$

$= l^2 - \frac{l^2}{3}$

$= \frac{2l^2}{3}$

$\Rightarrow ME = \sqrt{\frac{2l^2}{3}}$, so $ME = \frac{\sqrt{2}}{\sqrt{3}} l$. Finally, rationalise.

Answer: $\frac{\sqrt{6}}{3} l$

(7 marks)